Abstract

We investigate the effects of investment taxes and Tobin tax on capital flows in global markets. An optimization model is proposed to maximize the expected net Sharpe ratio and find the optimal risky portfolio across tax jurisdictions (namely, source only, residence only, mixed with credit, and mixed with double taxation). The experimental results show that a source only tax union has more capital transits in global markets than a residence only tax union, and its optimal market portfolio is more sensitive to regional tax policy. Tobin tax can reduce market volatility but the effect varies with its rate, certain market specifications, and investment tax rules. This effect does not depend on which side of the capital flow (inflow or outflow) is subject to Tobin tax. Finally, an agreement between countries on a Tobin tax rate (policy) can significantly reduce the negative effect of Tobin tax on capital flows.

Keywords: Tobin Tax; Nonlinear programming; Capital flows; tax policy
1. Introduction

The extant literature on the macro-level impact of tax can be divided into two main categories. One is the analysis of government tax revenues (Saunoris and Payneb, 2010; Creedy and Sanz-Sanzb, 2011). The other, to which this paper relates, concerns international tax-driven capital flows, and often compares the two main withholding tax systems: source-based and residence-based. In a source-based tax system, all investments in a country are taxed only by the government of that country, regardless of the residency of the investor. In a residence-based tax system, all investments are taxed only by the investor’s country of residence regardless of the investment location.

A comparison between these two tax systems can be made by observing the change in the two components of a country’s balance of payments, namely the external current account and the financial account. A current account surplus increases a country’s net foreign assets whereas a current account deficit reduces them. Bovenberg (1992) explores how residence- and source-based taxes on capital income affect the external current account in small open economies. This effect is examined indirectly using the identity between the external current account balance and the difference between domestic saving and domestic investment. The same method is also employed by Summers (1988) and Bovenberg (1989) in their discussion of the two tax systems. However, these papers explore the impact of two different tax systems on the saving-investment balance while income tax is isolated from all other taxes. In this paper, we rectify this shortcoming and combine income tax with capital gains tax as well as Tobin tax for the purpose of investigating the impact of tax policy within a more realistic tax
system.

Analysis of the current and financial accounts balance can help in understanding the general impact of the two tax systems on international capital flows, but it is difficult to deduce their impact on consumption as opposed to investment. Little research has been done on the impact of the two tax systems on global consumption. Some recent research, however, focuses on the comparison between source- and residence-based taxes for investment, generally examining the differential impact of the two tax systems on the global capital allocation of real industry investments (Devereux et al., 2008; Devereux and Griffith, 2003; Fuest and Huber, 2004; Fuest et al., 2005). Devereux et al. (2008) test whether The Organization for Economic Co-operation and Development (OECD) countries “compete with each other over corporate taxes in order to attract investment”. They conclude that countries compete over the effective average rate of tax and the statutory rate of tax, which reflects governments’ belief that international firms’ choices of location are discrete. Nevertheless, focusing on the impact on mergers and acquisitions together with Greenfield investments, Becher and Fuest (2011) reach a different conclusion. They highlight that in previous work, the model considers Greenfield investment only and neglects the large part of international capital flows that takes the form of mergers and acquisitions (M&A). Taking into account M&A investment leads to substantial changes in the efficiency properties of taxation. A similar conclusion is reached in other articles using different assumptions for the M&A market (Desai and Hines, 2004; Becker and Fuest, 2008, 2010).

Financial market investments account for a large part of global investments and
should also be considered when comparing the two tax systems, but this topic has not been fully investigated. There are studies on the impact of tax on financial markets but these focus on other tax issues. Some consider general transaction tax (Campbell and Kenneth, 1994; Edwards 1992; Hubbard, 1993), and others “Tobin” tax (Tornell, 1990; Reinhart, 1991). In this paper, we fill this gap by exploring the quantitative impact of international tax on financial market investments. The investment process is simulated with tax constraints that include income, capital gains and Tobin tax.

The idea of a Tobin tax was introduced by James Tobin in the early 1970s (Tobin, 1978). It has been controversial among economists and politicians ever since (Haq et al., 1996; Habermeier and Kirilenko, 2003; Weaver et al., 2003). In 2013, the European Commission officially announced that a tax on financial transactions out of or into 11 EU countries would be introduced in 2014. The countries involved are Austria, Belgium, Estonia, France, Germany, Greece, Italy, Portugal, Slovakia, Slovenia and Spain. However, this proposal is still under discussion and has been postponed. The discussions are concerned with two matters. The first is about the impact of Tobin tax on market efficiency (specifically, the deviation of asset prices from their fair values which, in articles on Tobin tax, is usually measured by price volatility). The other concerns the impact on trading volume.

As regards market efficiency, some articles conclude that Tobin tax improves market efficiency by decreasing price volatility (Frankel, 1996; Pally, 1999; Ehrenstein, 2002; Westerhoff, 2003; Ehrenstein et al., 2005; Cipriani and Guarino, 2008) while others conclude that Tobin tax reduces market efficiency by increasing price volatility.
(Kupiec, 1995; Aliber et al. 2003). The results are mainly derived from heterogeneous research methods and tax settings, for example the type of investors (long-term or short-term; speculators, fundamentalists or noise traders) and their motivation for trading. Through an experimental analysis on an artificial market with four types of trader, Mannaro et al. (2008) find that market efficiency decreases and price volatility increases. In contrast, Pally (1996) proposes a microeconomic model with two groups of risk-neutral traders (fundamentalists and noise traders) and shows that noise traders (speculators) create inefficiencies and higher costs for fundamentalists. He concludes that even though a Tobin tax would apply equally to noise traders and fundamentalists for a single trade, the overall impact is larger on noise traders as they trade more frequently. Consequently, noise trading would be reduced by a Tobin tax and market efficiency would be enhanced, which is contrary to the view expressed by Mannaro et al. (2008). There is no consensus, therefore, on the consequences of a Tobin tax on price volatility and market efficiency, although Haberer (2006) may help explain these apparent contradictions. He advocates a U-shaped relationship between market trading volume and price volatility, and concludes that market volume is reduced by a Tobin tax, but this new tax can have different impacts on price volatility, depending on the level of trading in the market.

As regards work on trading volume, all articles conclude that the introduction of Tobin tax would reduce trading volume by decreasing the transactions carried out by speculators (Haq et al., 1996; Weaver et al., 2003; Mannaro et al., 2008; Hanke et al., 2010). However, the analysis of Hanke et al. (2010) shows that, although a Tobin tax
reduces market trading volume, the size of this reduction is heterogeneous and highly sensitive to the size of the market. They present an experiment with currency trading on two artificial markets, in which none, one, or both markets have a Tobin tax. Their conclusion is that trading volume and trading activity are significantly affected if the Tobin tax is levied on the larger market, and the stronger influence of the tax on the larger market seems to be driven by drying up the hitherto very liquid large market.

Some further questions arise from the work of Hanke et al. (2010). For example, is the impact of the Tobin tax on trading volume also sensitive to other factors such as market liquidity and market correlation? In the literature, analysis of Tobin tax is isolated completely from other tax issues (e.g. income tax and capital gains tax). Is the impact of the Tobin tax on trading volume also sensitive to investment tax rules? When discussing Tobin tax, a model should also take into account income and capital gains taxes if investors are subject to them.

The above papers on Tobin tax explore its impact on individual trading activity (micro) rather than on government policy (macro). In this paper we consider simulating market movement under different international tax policies. This should help governments and policy setters better understand the advantages and disadvantages of Tobin tax.

We set up an artificial global financial market along the lines of Mannaro et al. (2008), and use it to investigate the impact of integrating Tobin tax with specific income tax and capital gains tax rules. The artificial market includes three regional sub-markets: the Eurozone, the U.K. and the U.S., which are currently in an international residence-
based tax union. We also allow tax rates to vary (the rate of Tobin tax is set in a range of 0% to 1% while the rate of the investment tax is set in the range of 10% to 70%).

An optimization model is developed to test the differential effects of source- and residence-based tax systems on financial market investments. In this manner, keeping all else the same, we can investigate whether a change from source-based tax to residence-based tax and joining the international tax union would benefit a developing country which is currently applying a source-based tax system. The setup also allows us to investigate whether a change from a residence-based tax union to a source-based tax union would improve the efficiency of developed markets. Finally, the differential impact of the Tobin tax under heterogeneous investment tax rules (both tax rates and withholding tax applications) can be tested.

2. Capital Asset Pricing Model with heterogeneous tax rules

Trauring (1979) develops a three-fund separation theorem leading to a three-term Capital Asset Pricing formula. The investor’s optimal risky portfolio is a linear combination of three identified risky portfolios, $G$, $D$, and $E$, which are independent of investors, their utility function, and tax brackets. This independence does not, however, extend to the weights for each identified risky portfolio. These weights are functions of investor $i$’s income tax and capital gains tax brackets, and will be denoted by $a_i$, $b_i$, and $c_i$. As a result, investors with different tax brackets will have different

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1 The investment tax includes income tax and capital gains tax.
2 Detailed definition of these three independent risk portfolios can be found in Trauring (1979).
optimal risky portfolios, \( Y_i \). Trauring presents the following formula to calculate the market portfolio with tax:

\[
\sum_{i=1}^{n} X_i = M = (\sum_{i=1}^{n} a_i) \ast (G\Gamma^{-1}) + (\sum_{i=1}^{n} b_i) \ast (D\Gamma^{-1}) + (\sum_{i=1}^{n} c_i) \ast (E\Gamma^{-1})
\]  

(1)

where \( M \) is the so-called market portfolio, \( X_i \) is optimal portfolio for investor group \( i \), \( \Gamma \) is the a matrix whose \( kj^{th} \) element is the ratio of covariance between assets \( k \) and \( j \) to the variance of the market, and \( n \) is the number of investor groups. He further proves that the capitalization-weighted sum of all investors’ optimal risky portfolios provides the market equilibrium condition. In Trauring (1979), however, the taxation process is still highly simplified. An asset’s net return is calculated as \((1-t)\ast r\) where only fixed tax rates \((t)\) are considered while more realistic and complex tax rules (e.g., annual income tax, deferred capital gains tax, withholding tax on foreign investment, and transaction tax) are not considered. In this paper, Trauring’s work is improved by including more complex and realistic tax settings in the mathematical programming.

An optimization model is proposed that considers heterogeneous tax rules across investors. To keep the model tractable, all investors are divided into only three residence groups: U.K., Eurozone and U.S. Investors in the same group are assumed to be subject to the same tax rules and hold the same optimal risky portfolio. To determine the market portfolio under taxation, the capitalization-weighted sum of the three local optimal risky portfolios is calculated. Thus:

\[
Portfolio_{Market} = (\sum Capitalization_i \times Portfolio_i) / \sum Capitalization_i
\]

(2)

In equation (2), \( Capitalization_i \) is the total market capitalization of regional market \( i \). \( Portfolio_i \) is the local optimal risky portfolio for regional market \( i \). The total
regional market capitalizations are then used as weights to calculate the weighted average optimal portfolio for the global market.

As in Trauring (1979), the market portfolio M is not a unique portfolio that all investors hold. In fact, it is a weighted sum of the optimal portfolios $X_i$ of different investor groups. Nevertheless, it can still be used to provide the composition of the global risky financial market if all investors with different tax brackets hold their own optimal portfolio and are included in the calculation of the aggregated market portfolio.

3. Tax rules

This section outlines the tax set-up adopted. Basic investment tax (i.e., annual income tax and tax on realized capital gains) is introduced together with heterogeneous Tobin tax and foreign investment (withholding) tax rules.

First, the basic investment tax rules are set out. Each asset class (bonds, equities and commodities) in each country or region (U.K., U.S. and Eurozone, treating the Eurozone as a single country) is subject to an independent income tax rate and capital gains tax rate, which may differ across asset classes and countries. With regards to income tax for bonds and equities, all income is assumed to be paid and taxed annually, and all net income is received in cash and can be used to purchase assets freely. Income tax is calculated and paid by asset, not by account. With regard to capital gains tax for bonds, equities and commodities, a single-period model is used. All capital gains tax

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3 Account: all assets subject to the same tax rules are put in one group (named as account) and are subject to the same tax constraints, e.g., U.K. bonds, U.S. equities, ..., etc.
can be deferred if the holding assets are not sold or ‘disposed of’ and withdrawals from a holding asset are subject to an instant capital gains tax payment (withdrawal tax) during the rebalancing process. However, these deferred taxes will be calculated and deducted from total return to get net return at the end of the period in the model. The calculation is in respect of each individual asset, not for the whole account. All the tax rates are initially set at 40%, the middle of the range (also the average tax rate historically for individuals in the U.K. and the U.S.; see KPMG’s website), but are allowed to vary between 0% and 100%. All returns from the risk-free asset (i.e., 3-month treasury bills), whether in the form of income or capital gains, are assumed to be free of tax.

Source-based taxation is justified on the grounds that the country which provides the opportunity to generate income or profits should have the right to tax it. In a source-based tax union, all investors, regardless of their residence, are subject to the same tax rules globally. The tax rate on an investment is only dependent on the asset class (e.g., U.S. equities or Eurozone bonds). Residence-based taxation is justified on the grounds that people and firms should contribute towards the public services provided by the country where they live, on all their income regardless of the location of its source. In a residence-based tax union, the tax rate on an investment is not only dependent on the asset class but also the investor’s country of residence. All foreign investments are taxed

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4 In this work, the withdrawal tax is the same as capital gains tax. All withdrawals from the sale of assets are subject to capital gains tax.

firstly by the local (foreign) government, and any positive gap between the home and local tax rates will be paid later to the government of the investor’s country of residence.\(^6\) (Note: a positive gap means that the home tax rate is higher than the local tax rate.) If the gap is negative, no further tax is payable to the government of the investor’s country of residence.

Consider two countries in a mixed international tax environment, one adopting a residence-based tax system and the other adopting a source-based tax system. Two different methods are required to calculate the effective tax rate: the credit method and the double taxation method.

In the experiments, all three regions (U.K., U.S. and E.U.) are initially assumed to be a residence-based tax system, which conforms to the present situation. However, some of them are then replaced by a source-based tax system to investigate the impact of heterogeneous withholding tax on market performance.

Finally, when capital moves from one country to another, a Tobin tax may be payable. All transfers within one country are not subject to the Tobin tax. In the model, three different Tobin tax rules are investigated: ‘inflow tax only’ whereby investors are required to pay the Tobin tax to the country if and only if they transfer capital into it; ‘outflow tax only’ whereby investors are required to pay the Tobin tax to the country if and only if they transfer capital out of it; and ‘two-side tax’ whereby investors are required to pay the Tobin tax to the country if they transfer capital either into it or out

\(^6\) In this work, we define: (1) Home country: the investor’s country of residence. (2) Local country: the country of investment. (3) Overseas country: the country other than the investor’s country of residence
of it. Some other assumptions are also applied.\textsuperscript{7}

4. Problem constraints and objective functions

4.1. Basic trading constraints

The notation used in the following exposition is explained in Table 1 in the Appendix.

4.1.1 Local trading budget

A local trading budget (balance) constraint ensures that for all accounts, $k=1, 2, 3$ (note: 1 represents commodities, 2 represents bonds and 3 represents equities), in the same country $j$, $j=1, 2, 3$ (note: 1 represents the U.K., 2 represents the U.S. and 3 represents the Eurozone), the total selling proceeds from the local market, $1' l^s_{kjm}$, and the total external funding for the local investments, $C_0$, are equal to the total buying costs within the local market, $1' l^b_{kjm}$, so that:\textsuperscript{8}

$$
\sum_k 1' l^b_{kjm} = \sum_k 1' l^s_{kjm} + C_0; \quad j, m = 1, 2, 3
$$

All local transits within the same country or region are not subject to a Tobin tax. ‘$m’’ represents the group of investors who have the same country of residence (note: 1

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\textsuperscript{7} The following are also assumed: all investors have the same available international asset classes; there is no exchange rate risk; total global wealth is constant; the regional supply of asset units is constant (i.e., there are no stock splits, rights issues or other new issues); an asset’s demand increases if its weight in the aggregated market portfolio is higher; and the asset supply is inelastic so a percentage change in asset weights corresponds to the percentage change in capitalization; international tax is simplified (ignoring annual exemption allowances); ‘Eurozone bond’ is index of French and German corporate and government bonds; ‘Eurozone’ uses one tax rate for one asset class; no capital flow restriction or friction; single period model; only consider risky assets; ‘assets’ are not securities but investible asset classes.

\textsuperscript{8} This budget includes all rebalancing activity within a single country, and we assume different asset classes are traded within different accounts.
represents U.K. residents, 2 represents U.S. residents and 3 represents Eurozone residents).

4.1.2 International trading budget

An international trading budget (balance) constraint ensures that for all accounts in all countries, the total international selling proceeds, $1'i^s_{kjm}$, and the total external funding for international transits, $C_{0m}$, are equal to the total international buying costs, $1'i^b_{kjm}$, so that

$$\sum_{k,j} 1'i^b_{kjm} = \sum_{k,j} 1'i^s_{kjm} + C_{0m}; \ m = 1, 2, 3$$  \hfill (4)

All international transits between different countries are subject to Tobin tax.\(^9\)

4.1.3. Diversification and maximum holdings

A diversification constraint is formulated by setting an upper bound on the value of each asset in a portfolio. Thus,

$$w^{jm}_{k1} \leq U * \delta_{kjm} \ \forall k, j, m = 1, 2, 3,$$  \hfill (5)

$$0.0001 * \delta_{kjm} \leq w^{jm}_{k1} \ \forall k, j, m = 1, 2, 3,$$  \hfill (6)

where $U$ is the maximum holding weight for a single asset and is set equal to 0.05 in the optimization. If investors do not want to hold an asset in the new portfolio, the corresponding variable, $\delta_{kjm} \in [0,1]$ in (6) must be equal to zero. If investors want to hold an asset in the new portfolio, the corresponding variable $\delta_{kjm}$ in (5) will be non-zero. By also setting a lower bound on the total number of assets in a portfolio, $N_{min}$, the firm-specific (or industry-specific) risk can be minimized in the market portfolio.

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\(^9\) This budget includes all rebalancing activities across countries, rebalancing investments from one country to another.
\[ \sum_{k,j} 1' \delta_{km} \geq N_{min}, \ m=1, 2, 3, \]  
(7)

where the sum of the variables, \( \delta_{km} \in [0,1] \), counts this number. We set \( N_{min} \) at 20 in this work.

4.2. Taxation

The total tax liability is built up by calculating the impact of different tax rules on the cumulative returns, withdrawals and wealth.

4.2.1. Source- and residence-based tax systems

In a residence-based tax union, all investments are taxed by the country \( j \) in which the investment is made first, at rate \( t_{cg}^{kj} \) on capital gains and \( t_{in}^{kj} \) on incomes. If the investor is resident in a different country \( m \) with higher rates of tax \( t_{cg}^{km} \) and \( t_{in}^{km} \), the gap will be taxed later. So the effective rate of tax on asset \( k \) in country \( j \) for an investor from country \( m \) is always the higher rate in the two countries on asset \( k \).

\[
t_{cg}^{km} = \max\{t_{cg}^{kj}, t_{cg}^{km}\}, \quad t_{in}^{km} = \max\{t_{in}^{kj}, t_{in}^{km}\}
\]
(8)

In a source-based tax union, all investments are taxed by the invested country only.

So the effective rate of tax is always the rate in the invested country.

\[
t_{cg}^{km} = t_{cg}^{kj}, \quad t_{in}^{km} = t_{in}^{kj}
\]
(9)

In a mixed tax system, the taxation between two countries both under a residence-based tax system remains the same as previously. However, there are two different methods for calculating taxation between countries under different tax systems.

i. Credit method

The country with a source-based tax system is assumed to have a tax agreement
with the other two countries. So if the country in which the investment is made (the
‘invested’ country) implements a source-based tax system, while the investor is from a
different country with residence-based tax system, the effective tax rates are:

\[ t_{cg}^{km} = \max\{t_{cg}^{kj}, t_{cg}^{km}\}, t_{in}^{km} = \max\{t_{in}^{kj}, t_{in}^{km}\} \] (10)

where \( t_{cg}^{km} \) is capital gains tax rate for account of class k and residence country of
group m, while \( t_{in}^{km} \) is annual income tax rate for account of class k and residence
country of group m. If the invested country implements a residence-based tax system
while the investor is from a different country with a source-based tax system, the
effective tax rates are:

\[ t_{cg}^{km} = t_{cg}^{kj}, t_{in}^{km} = t_{in}^{kj} \] (11)

ii. Double tax method

The country with a source-based tax system is assumed to have no tax agreement
with the other two countries. So if the invested country implements a source-based tax
system while the investor is from a different country with a residence-based tax system,
the investment is taxed by both countries.

\[ t_{cg}^{km} = 1 - (1 - t_{cg}^{kj})(1 - t_{cg}^{km}), t_{in}^{km} = 1 - (1 - t_{in}^{kj})(1 - t_{in}^{km}) \] (12)

In equation (12), \( (1 - t_{cg}^{kj})(1 - t_{cg}^{km}) \) is the rate of capital gains remaining after tax
payment by both countries. The effective tax rate for capital gains \( t_{cg}^{km} \) is then
calculated as 1 minus this remaining rate. A similar method is applied to obtain the
effective tax rate for income.

If the invested country implements a residence-based tax system while the investor
is from a different country with a source-based tax system, the effective tax rates are:
\[ t_{cg}^{kjm} = t_{cg}^{kj}, t_{in}^{kjm} = t_{in}^{kj} \]  

(13)

4.2.2. Cumulative capital gains

The remaining capital gains available for withdrawal in each account are calculated as:

\[ R^{1m}_{11} = R^{1m}_{10} - \frac{x_{1jm}^2}{1 - t_{cg}^{1jm}} - X_{1jm}^2/(1 - t_{cg}^{1jm}) \]  

(14)

\[ R^{1m}_{21} = R^{1m}_{20} - \frac{x_{2jm}^2}{1 - t_{cg}^{2jm}} - X_{2jm}^2/(1 - t_{cg}^{2jm}) \]  

(15)

\[ R^{1m}_{31} = R^{1m}_{30} - \frac{x_{3jm}^2}{1 - t_{cg}^{3jm}} - X_{3jm}^2/(1 - t_{cg}^{3jm}) \]  

(16)

Here, \( x_{kjm}^2 \) represents the net amounts of cash withdrawal from the capital gains of each asset for international transits, while \( X_{kjm}^2 \) represents the net amounts of cash withdrawal from the capital gains of each asset for local transits. There are two sources of withdrawal: capital gains and initial capital. The difference represents the cash from the initial capital \( x_{kjm}^3 \) and \( X_{kjm}^3 \), which is free of tax at encashment, while the cash in \( x_{kjm}^2 \) and \( X_{kjm}^2 \) are subject to an immediate tax payment. In addition, \( R^{1m}_{k0} \) is the previously cumulated unrealized capital gains, and an upper bound on the total withdrawals is set at \( R^{1m}_{k1} \geq 0 \) (\( \forall k, j, m = 1,2,3 \)).

4.2.3. Withdrawals

When an asset is sold (or withdrawn), the gross amount from unrealized capital gains \( (x_{kjm}^2, X_{kjm}^2) \), which is proportional to the total gross amount \( (t_{kjm}^s/(1 - t_{j}^{out}), t_{kjm}^s) \), needs to be calculated first so that withdrawal tax on this amount can be calculated.

\[ \forall k = 1,2,3; \forall j = 1,2,3 \]
\[ x_{kjm}^2 = ((1 - t_{cg}^{kjm})R_{k0}^{jm}/(w_{k0}^{jm} - t_{cg}^{kjm} \ast R_{k0}^{jm})) \circ i_{kjm}^s / (1 - t_j^{out}) \] (17)

\[ x_{kjm}^3 = i_{kjm}^s / (1 - t_j^{out}) - x_{kjm}^2 \] (18)

\[ X_{kjm}^2 = ((1 - t_{cg}^{kjm})R_{k0}^{jm}/(w_{k0}^{jm} - t_{cg}^{kjm} \ast R_{k0}^{jm})) \circ l_{kjm}^s \] (19)

\[ X_{kjm}^3 = l_{kjm}^s - x_{kjm}^2 \] (20)

Since withdrawals for local transits are all exempt from Tobin tax, the term \((1 - t_j^{out})\) is not present in constraints (19) and (20).

4.2.4. Wealth

We next calculate the total wealth in each account after trading and at the end of the period. In calculating the former, the transactions between assets are considered.

\[ w_{11}^{jm} = w_{10}^{jm} - \left( \frac{x_{1jm}^2}{1 - t_{cg}^{1jm}} + x_{1jm}^3 \right) - \left( \frac{X_{1jm}^2}{1 - t_{cg}^{1jm}} + X_{1jm}^3 \right) + (1 - t_j^{in})i_{1jm}^b \] (21)

\[ w_{21}^{jm} = w_{20}^{jm} - \left( \frac{x_{2jm}^2}{1 - t_{cg}^{2jm}} + x_{2jm}^3 \right) - \left( \frac{X_{2jm}^2}{1 - t_{cg}^{2jm}} + X_{2jm}^3 \right) + (1 - t_j^{in})i_{2jm}^b \] (22)

\[ w_{31}^{jm} = w_{30}^{jm} - \left( \frac{x_{3jm}^2}{1 - t_{cg}^{3jm}} + x_{3jm}^3 \right) - \left( \frac{X_{3jm}^2}{1 - t_{cg}^{3jm}} + X_{3jm}^3 \right) + (1 - t_j^{in})i_{3jm}^b \] (23)

We distinguish between a transit within the same country and a transit between two countries. Tobin tax is applied to the latter transit only. In calculating wealth at the end of the period, both capital gains and incomes are considered, and the corresponding annual income tax payments and management fees are deducted (management fees are set as a parameter which can be either zero or positive).
\[ w_{12}^{jm} = (1 - f_{1j})[(1 + \bar{g}_{1j}) \circ w_{11}^{jm}] \]  
\[ w_{22}^{jm} = (1 - f_{2j})[(1 + \bar{g}_{2j}) \circ w_{21}^{jm}] \]  
\[ w_{32}^{jm} = (1 - f_{3j})[(1 + \bar{g}_{3j}) \circ w_{31}^{jm}] \]

The income return should also be calculated and included in the total wealth.

\[ C_{2m} = \sum_{k,j} (1 - f_{kj})[(1 - t_{in}^{km}) (\bar{d}_{kj}' w_{k1}^{jm})] \]

Here, \( k = 2, 3 \) only since there is no income from commodities.

4.2.5. Cumulative capital gains tax

Finally, the total tax liability is calculated by adding deferred tax from previous periods to that of the current period. Since this is a single-period model, all previous tax liabilities will have already been determined.

\[ T_{10}^{jm} = T_{10}^{jm} + t_{cg}^{1jm} (1 - f_{1j}) (\bar{g}_{1j}' w_{11}^{jm}) - \{t_{cg}^{1jm} / (1 - t_{cg}^{1jm})\} \]  
\[ T_{20}^{jm} = T_{20}^{jm} + t_{cg}^{2jm} (1 - f_{2j}) (\bar{g}_{2j}' w_{21}^{jm}) - \{t_{cg}^{2jm} / (1 - t_{cg}^{2jm})\} \]  
\[ T_{30}^{jm} = T_{30}^{jm} + t_{cg}^{3jm} (1 - f_{3j}) (\bar{g}_{3j}' w_{31}^{jm}) - \{t_{cg}^{3jm} / (1 - t_{cg}^{3jm})\} \]  
\[ T_{k0}^{jm} = (1' R_{k0}^{jm}) t_{cg}^{km} \]

In equation (28), \( t_{cg}^{1jm} (1 - f_{1j}) (\bar{g}_{1j}' w_{11}^{jm}) \) is the tax on the capital gains for the current period after management fees. This amount is then added to total cumulative tax \( T_{10}^{jm} \). As \( 1' x_{1jm}^2 \) is the net amount after tax, the expression \( \{t_{cg}^{1jm} / (1 - t_{cg}^{1jm})\} \] is used to calculate the tax payment towards withdrawal for international transits. A similar method is used to calculate the tax payment towards
withdrawals for local transits.

The final net return for each account is obtained by subtracting all the contingent capital gains tax from the account:

\[ TR_{kjm} = 1'w_{k2}^{im} - T_{k2}^{jm} \quad \forall k, j, m = 1, 2, 3 \quad (32) \]

4.3. After-tax market portfolio

In the CAPM, it is assumed that all investors are rational and are expected to hold the same market portfolio of risky assets (usually proxied by a comprehensive ‘Index’), which maximizes the portfolio expected excess return over a risk-free rate per unit of portfolio risk (i.e., the Sharpe ratio). In this paper, the objective function is the after-tax Sharpe ratio. For each group of investors, \( m = 1, 2, 3 \), an independent risk-free rate of return on an after-tax basis is introduced. This is because investors from different countries have access to a different risk-free rate of return. The optimization models are used to find the optimal risky portfolio for each group of investors who have the same country of residence.

For group \( m = 1, 2, 3 \),

Maximize \( \left( \sum_{k,j} TR_{kjm} + C_{2m} - R_{f}^{m} \right)/\sqrt{w_{1m}^{1}w_{1m}^{2}} \)

Subject to: Constraints: If residence-based (3)–(8),(14)–(32);

If source-based (3)–(7),(9),(14)–(32);

If mixed credit method (3)–(7),(10)–(11),(14)–(32);

If mixed double taxation (3)–(7),(12)–(32);

\[ TR_{kjm} \geq 0, k = 1, 2, 3, j = 1, 2, 3; \quad T_{k2}^{jm} R_{k2}^{jm} \geq 0, k = 1, 2, 3, j = 1, 2, 3; \]
\[ w_{k1}, l_{kjm}, l_{kjm}^b, l_{kjm}^s, k = 1, 2, 3, j = 1, 2, 3; \]
\[ x_{kjm}^2, x_{kjm}^3, X_{kjm}^2, X_{kjm}^3 \geq 0, k = 1, 2, 3, j = 1, 2, 3; \]
\[ \delta_{kjm} \in [0, 1], k = 1, 2, 3, j = 1, 2, 3; \]

where \( R_f \) is the expected total wealth from investing in the risk-free asset.

\[ w_{1m}^r \sum w_{2m}, \text{ where } w_I \text{ is a vector } (w_{1m}^{11}, w_{2m}^{11}, w_{3m}^{11}, w_{1m}^{21}, w_{2m}^{21}, w_{3m}^{21}, w_{1m}^{31}, w_{2m}^{31}, w_{3m}^{31}) \text{ of all the asset weights in period } 1 \text{ (end of rebalancing), is equal to the variance } \sigma^2 \text{ of the portfolio.} \]

After obtaining the local optimal risky portfolio for each group of investors, we calculate the market portfolio using the following formula:

\[
\text{Portfolio}_{\text{market}} = \frac{\sum \text{Capitalization}_m \times \text{Portfolio}_m}{\sum \text{Capitalization}_m}
\]

(Note: Henceforth, we assume the weight for Eurozone and U.S. investors is 0.4, and the weight for U.K. investors is 0.2.)

5. Influence of taxation on portfolio management

5.1. Data, figures and experimental method

In the optimization, each equity, bond and commodity segment of the targeted market is divided into several subclasses. Commodities are based in all three markets (Eurozone, U.K. and U.S.) and are categorized by product type (e.g., oil, gold, copper, corn … etc.). All bonds currently available in the market are first divided into two subclasses. Commodities based in all three markets (Eurozone, U.K. and U.S.) and are categorized by product type (e.g., oil, gold, copper, corn … etc.).

10 The weight for the E.U. investors here is the ratio of the total capital held by investors from the E.U. to the total capital held by all investors. The weight for the U.K. and the U.S. investors is derived in the same way.
groups: investment grade and high yield. Each of these groups is further divided into industrial subclasses (airline, technology, telecommunications … etc.). Equities are also categorized by industry sector in all three markets. This generates 18 classes of U.K. commodities, 20 of U.S. commodities, 20 of Eurozone commodities, 7 of U.K. bonds, 24 of U.S. bonds, 24 of Eurozone bonds (mainly German and French bonds), 30 of U.K. shares, 40 of U.S. shares and 30 of Eurozone shares.

All the historical annual dividends and capital gains of the asset classes are obtained from Datastream for the period 1990 to 2011. Data for commodities are from the S&P commodity index. Data for bonds are from Barclay’s bond index, including both government and corporate bonds. Data for equities are from the FTSE for the U.K., US–DS Price Index for the U.S., and the FTSEUR1ST 300 for the E.U. Capital gains for each asset class are calculated as the change of index prices and adjusted by excluding corresponding income. After obtaining the optimal portfolio, investors can then invest in the corresponding index by purchasing index futures or holding the assets in each index directly.

5.2. Residence-based and source-based tax systems

In the experiments, investment tax rates for all asset classes are set initially at 40%. Then, in turn, each is changed incrementally from 10% to 70% so that the change of...
the optimal portfolio, and therefore the capital flow due to the change of tax rates, can be observed. In addition, it is assumed that the U.K., the U.S. and the Eurozone investors represent 100% of global markets. The proportion of total wealth of U.K. investors is taken to be 20%, while the proportion of total wealth of both Eurozone and U.S. investors is taken to be 40%.

In Figure 1, $a = \frac{R}{w}$. This is the ratio of cumulative return to total asset weights. A higher ratio here means more capital gains tax remaining for the payment at encashment and, therefore, requires a higher expected return to rebalance the portfolio. The three charts show how the optimal weight of the local market varies with the local investment tax rate under different withholding tax systems. In each chart, the curve ‘Residence Only’ is obtained when all three regions apply residence-based withholding tax and are in an international tax union. Both ‘Mixed (Credit Method)’ and ‘Mixed (Double Taxation)’ curves are obtained when the Eurozone and the U.K. apply residence-based withholding tax while the U.S. applies source-based withholding tax. However, the former assumes all three regions are still in an international tax union (Credit Method) while the latter assumes only the Eurozone and the U.K. are in a tax union (Double Taxation). The ‘Source Only’ curve is obtained when all three regions apply a source-based investment tax. In this tax system, all income is subject to a tax payment only in the country in which it is generated.

5.2.1. Residence only tax system

It can be seen from all three charts in Figure 1 that, for all three regional markets (Eurozone, U.K. and U.S.), the change of the market weight with the ‘Residence Only’
tax system is usually the smallest in comparison to the other three tax systems: ‘Source Only’, ‘Mixed (Credit Method)’ and ‘Mixed (Double Taxation)’. From the second chart, the weight of the U.K. market only changes from 60% to 62% as its investment tax rate is cut from 40% to 30% with the ‘Residence-Only’ tax systems. This is much smaller than the changes with other tax systems, which are from 60% to 76% for ‘Source Only’, from 60% to 79% for ‘Mixed (Credit Method)’ and from 70% to 90% for ‘Mixed (Double Taxation)’. In addition, the difference in the change of market weight is usually obvious for ‘Residence Only’ tax, particularly in comparison to ‘Source Only’ tax. From the first chart in Figure 1, as the tax rate is cut from 40% to 10%, the weight of the Eurozone market increases from 30% to over 90% with ‘Source-Only’, while the weight increases only to 58% with ‘Residence Only’.

The experimental results also show that with ‘Residence Only’ tax, the change as the tax rate is cut is usually smaller than the change as the tax rate is increased. For example, from the third chart in Figure 1, the weight of the U.S. market rises from 21% to just 30% as the tax rate is cut to 10% while the weight decreases to almost 0% as the rate is increased to 70%. This is because with ‘Residence Only’ tax, the change in market weight is mainly due to rebalancing by local investors only when the rate falls below 40%, while the change is due to rebalancing by overseas investors as the rate increases above 40%. Under a ‘Residence Only’ tax system, when a regional market’s

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13 Local investor: the investor whose country of residence is the country we are discussing (or so-called focused market).
14 Local investor: the investor whose country of residence is the country we are discussing (or so-called focused market).
tax rate is cut below the other markets’ tax rate, only the local investor’s local investment\textsuperscript{15} obtains the relatively low tax rate\textsuperscript{16}. As a result, such a tax cut will only benefit local investors and leads to a capital inflow to the local market due to rebalancing by local investors. In contrast, when the local market increases the tax rate above 0.4, local investors will always be taxed at the higher rate wherever they invest. So the local investors are not motivated to rebalance.

5.2.2. Source only tax system

With a ‘Source Only’ tax union, changes in the local market weight are much larger than changes in the other three tax systems (‘Residence Only’, ‘Mixed (Credit Method)’ and ‘Mixed (Double Taxation)’). This higher sensitivity leads to greater volatility of markets when a regional market changes its tax rules. This is because with a source-based tax system, the change of tax in the local market will affect all investors’ investments in the local market and will lead to a large amount of rebalancing globally. On the other hand, this feature enables the regional government to intervene in the local market in extreme cases.

5.2.3. Mixed (Credit Method) tax system

The U.S. market is assumed to apply source-based taxation while the other two markets still apply a residence-based tax. We use this setting to test how a local tax change would affect global capital flow when heterogeneous withholding tax rules are

\textsuperscript{15} Local investment: an investment allocated to the focused market.

\textsuperscript{16} For example, when the Eurozone market’s tax rate is cut to 0.1 while the rate for the other two markets is still 0.4, only Eurozone investors’ investment in Eurozone assets are taxed at 0.1; all other investors’ investments are still taxed at 0.4.
used. In a ‘Mixed (Credit Method)’ tax system, we assume that the U.S. market is still in a tax union with the other two markets and double taxation is effectively eliminated.

It can be seen from the first two charts in Figure 1 that when the tax rate is cut below 40%, the change of market weight is usually larger in a ‘Mixed (Credit Method)’ tax system than in a ‘Residence Only’ tax system for the Eurozone and the U.K. markets. For example, from the second chart in Figure 1, the ‘Mixed (Credit Method)’ curve always stands above the curve for ‘Residence Only’ as the tax rate is cut below 40%.

In other words, the market will be more sensitive if a regional market with a source-based tax system is added to a ‘Residence Only’ tax union. This extra sensitivity is due to rebalancing by U.S. investors whose country of residence is assumed to apply a source-based tax system.

Consider a tax cut in the Eurozone and the U.K. markets, which are assumed to charge taxes based on investors’ country of residence in both ‘Residence Only’ and ‘Mixed (Credit Method)’ tax systems. In a ‘Residence Only’ tax union, as the tax rate is cut, as mentioned previously, only the local investments of local investors will be affected. In contrast, in a ‘Mixed (Credit Method)’ tax system, a tax rate cut in the Eurozone or the U.K. market will affect not only local investors but also U.S. investors. In a ‘Mixed (Credit Method)’ tax system, the U.S. market is assumed to use a source-based tax, and therefore the U.S. investors’ investment in the Eurozone or the U.K. market is only taxed at the Eurozone or the U.K. tax rate. So such a tax cut in a ‘Mixed (Credit Method)’ tax system will lead to a larger rebalance for US investors when they hold assets in the other two markets. This implies a larger capital flow (from US to
Eurozone and the U.K.) in global markets relative to the same cut in a ‘Residence Only’ tax system.

With regard to a tax increase in the Eurozone and the U.K. market, when the tax rate is set above 40% in the Eurozone or the U.K. market, the change of investment tax rate in a ‘Mixed (Credit Method)’ tax system will impose a similar effect on global markets as in a ‘Residence Only’ tax system. This is because the higher tax rate in a market using a residence-based tax system will increase the effective tax rate on the investment in that market to all investors regardless of whether it is in a ‘Residence Only’ tax system or a ‘Mixed (Credit Method)’ tax system. So markets will have the same sensitivity to the tax rate change in both tax systems.

In addition, we look at a tax rate change in the U.S. market, which uses a residence-based tax in a ‘Residence Only’ system rather than a source-based tax in a ‘Mixed (Credit Method)’ tax system. We find that a tax cut (from 40% to a lower rate) in the U.S. market will lead to the same rebalancing for U.S. investors (holding more U.S. assets) regardless of whether it is in a ‘Residence Only’ tax system or a ‘Mixed (Credit Method)’ tax system. On the other hand, a tax increase (from 40% to a higher rate) will lead to a greater rebalancing (holding more Eurozone and U.K. assets) and therefore larger capital flows (from the U.S. market to the other two markets) in a ‘Mixed (Credit Method)’ tax system rather than in a ‘Residence Only’ tax system.

In summary, if a market applies a source-based tax, it will be more sensitive to a tax increase than a tax cut. In contrast, if a market applies a residence-based tax, it will
be more sensitive to a tax cut than a tax increase. These results will be of interest to both governments and investors.

5.2.4. Mixed (Double Taxation) tax system

Again, the U.S. market applies source-based taxation while the other two markets still apply residence-based taxation. We can test whether a local tax change will affect global capital flows if heterogeneous withholding tax rules are used, and countries with different tax rules are not in a tax union. In Figure 1, the horizontal axis for all three charts is the investment tax rate (both income tax and capital gains tax) of the particular market and the vertical axis is the weight (or percentage) of that market to the global market in the obtained market portfolio. For example, if the weight for the Eurozone market in chart one is 20%, it means that the total summed weight of all Eurozone assets in the obtained market portfolio is 20%. The three charts in Figure 1 show that in a ‘Mixed (Double Taxation)’ tax system, on average, the weight of the Eurozone and the U.K. markets is higher (the curves stand above the curve of the U.S. market) while the weight on the U.S. market is lower throughout the whole tax rate range in comparison to ‘Residence Only’ and ‘Mixed (Credit Method)’ tax systems. This is because in a ‘Mixed (Double Taxation)’ tax system there is double taxation on investments in the U.S. market from overseas investors (Eurozone and U.K. investors) and, therefore, its effective tax rate is always higher than that in ‘Residence Only’ and ‘Mixed (Credit Method)’ tax systems. Double taxation will roughly lead to a 40% decrease in the initial U.S. market weight, which largely reduces the attraction of the market with source-based tax. Apart from the lower initial market weight, the shape of the ‘Mixed (Double
Taxation)’ curves is more or less the same as that for the ‘Mixed (Credit Method)’ in Figure 1.

Further experiments are now carried out by doubling the initial unrealized capital gains amount in the global market portfolio (i.e., the parameter \( a = \frac{R_{km}}{w_{km}} \) is changed from 0.2 to 0.4). First, the unrealized capital gains are doubled in only one market to obtain the first chart in Figure 2. Next, the unrealized capital gains in all markets are doubled, and the second chart in Figure 2 is obtained. The two charts of Figure 2 show the differential impact of Residence Only withholding tax on global markets with two distinct unrealized capital gains amount, ‘\( a \)’. In detail, in the first chart, we assume only one regional market’s ‘\( a \)’ is changed from 0.2 to 0.4. In the second chart, we assume all three regional markets’ ‘\( a \)’ are changed from 0.2 to 0.4. The horizontal axis of these two charts is still the investment tax rate while the vertical axis is the ratio of the regional market’s weight with \( a = 0.4 \) to its weight with \( a = 0.2 \). In summary, when the tax rate is cut from 0.4 to 0.3, a vertical axis value below 100% means a reduced amount of capital flowing into the particular market. Conversely, when the tax rate is raised from 0.4 to 0.5, a vertical axis value above 100% means a reduced amount of capital flowing out of that market. The two charts in Figure 2 show that when the unrealized capital gains make up a higher proportion of the holding assets’ value, keeping all else the same, the rebalancing amount is reduced. The second chart in Figure 2 shows that as the tax rate is cut, the local market’s weight decreases more when \( a = 0.4 \) than when \( a = 0.2 \). This means that increased unrealized capital gains in foreign markets would lead to lower capital flows into the local market with the same tax cut and reduces the ability of
governments to intervene in their local market using tax policy. On the other hand, it also shows that if an asset’s expected net return increases, more unrealized capital gains in the market reduces the rebalancing activity and therefore the volatility (trading volume) of the market. In addition, when only one market’s unrealized capital gains are doubled, the inflow of capital to this market will not be affected but the outflow of capital will be largely reduced (see the first chart in Figure 2). In contrast, when all markets’ unrealized capital gains are doubled, both the inflow and outflow of capital will be significantly reduced (roughly 50% on average, see the second chart in Figure 2). A trending market (i.e., the real value of the asset in the market increases in the long term) usually creates more unrealized capital gains than a volatile but non-trending market (i.e., the real value of the asset in the market remains the same in the long term, but its price moves around its constant real value). These results lead to the conclusion that in the long-term, increasingly more capital will flow from the volatile market to the trending market, and the volatile market must offer a higher return to maintain investment capital. This conclusion is consistent with rational investors who would require higher returns in riskier (volatile) environments.

5.3. Tobin tax

The rate of investment tax is now changed to simulate a change in the asset’s expected net return, and an investigation is carried out into how the introduction of heterogeneous Tobin tax affects the rebalancing of investors and therefore capital flows between regional markets. From the nine charts in Figs. 3 to 5, we can see that,
regardless of the withholding tax system, when the investment tax rate increases above
40%, the introduction of Tobin tax has a positive effect (i.e., it leads to an increase in
the optimal market weight by reducing capital outflows). In contrast, when the tax rate
is cut below 40%, Tobin tax has a negative effect (reduction in the optimal market
weight by preventing capital inflows). For example, in the first chart of Figure 3, for
the ‘$Tin\&Tout=1\%$’ where both inflows and outflows of capital are Tobin tax charged,
the curve increases above 100% as the investment tax rate is increased to 50% but
decreases below 100% as the investment tax rate is cut to 30%. In addition, whether
using a consistent Tobin tax rate globally ‘$Tin=0.5\%$’ improves market efficiency more
than when using different Tobin tax rates in different regions ‘$Tin=0;0.5\%;1\%$’ is
investigated.\textsuperscript{17}

From Figure 3, it can be seen that with the same change in asset expected returns,
investors’ rebalancing strategy, and consequently capital flows between markets, are
highly sensitive to Tobin tax. The third chart in Figure 3 shows that for the
‘$Tin\&Tout=0.5\%$’ curve, Tobin tax can reduce the flow from the rebalancing process by
20\text{-}40\%. In extreme cases, the Tobin tax can even reduce the flow by up to 44\% (see
the ‘$Tin\&Tout=1\%$’ curve in the third chart of Figure 3). This capital-lock effect is
heterogeneous across different markets and different tax rate changes (reduce the
amount of capital from both inflow and outflow.). For example, in the first chart of
Figure 3, as the investment tax rate is cut from 40\% to 30\%, Tobin tax ‘$Tin\&Tout=1\%$’
reduces the total capital inflows by 66\% for the U.K. market (the optimal weight of the

\textsuperscript{17} U.K.: $Tin=0$; U.S.: $Tin=0.5\%$; E.U.: $Tin=1\%$.}
U.K. market is only 60% of the weight as $Tin&Tout$ is changed from 0 to 1%). In contrast, in the second chart of Figure 3, as the investment tax rate is cut from 40% to 30%, the same Tobin tax $Tin&Tout=1\%$ reduces the total capital inflows by only 20% for the Eurozone market.

Figure 3 shows that the market capital flow will usually be much more sensitive to Tobin tax with a small change in tax rate (e.g., a change from 40% to 50% or a change from 40% to 30% leads to greater rebalancing activity than larger changes). For example, in the first chart of Figure 3, both the $Tin&Tout=1\%$ and $Tin&Tout=0.5\%$ curves reach their peak value as the investment tax rate is increased from 40% to 50%.

This is because a small change in the investment tax rate, and consequently a small change in an asset’s expected return, will give investors little motivation to rebalance. When the motivation is small, the cost of Tobin tax is a major concern and may exceed the extra return (benefit) obtained from rebalancing, so a Tobin tax will be important to investors’ optimal portfolios. In contrast, when the motivation is large, the cost of Tobin tax is relatively small and rebalancing is beneficial. Charging a Tobin tax in this situation will, therefore, have little influence on investors’ optimal portfolios. In conclusion, the market will be more sensitive to the implementation of Tobin tax in a trending market compared to a volatile market. From the government’s point of view, the Tobin tax will reduce its ability to intervene in the market. This reduction (up to more than 50%) varies by the investment tax rules (Resident Only, Source Only or others) applied by the governments. Therefore, the introduction of Tobin tax hinders economic policy changes.
Figure 3 shows that if the effective Tobin tax rate is similar, such as ‘$T_{in\&out}=0.5\%$’ and ‘$T_{in}=1\%$’, taxing both capital inflows and outflows and taxing only capital inflows or only capital outflows will have similar impact on markets. Thus, the impact of Tobin tax depends only on its effective rate but not on the flow of capital which is taxed. In detail, if major countries around the world decide to build a Tobin tax union, an agreement on the effective rate of Tobin tax will suffice. Individual countries can then tailor their own Tobin tax rules (charging on inflows or outflows or both) to their individual circumstances.

Comparing the three withholding tax settings: ‘Residence Only’, ‘Mixed (Credit Method)’, and ‘Source Only’ (Figs. 3 to 5), it is clear that if the same Tobin tax rule is applied, the shape of the curves for each regional market remains roughly the same no matter which withholding tax system is used. However, the peak value (or the volatility) of the curves is not the same. So the impact of Tobin tax on markets is also dependent on other tax rules, such as withholding tax rules. As a result, when a government tries to predict the market response to the introduction of Tobin tax, taxes other than Tobin tax must also be considered. Ignoring investment taxes or withholding tax in an investigation of Tobin tax will lead to an inaccurate prediction of its impact.

We test the impact of Tobin tax with different amounts of unrealized capital gains in the market. Similar to the experiment in section 5.2, we change the parameter ‘$a$’ from 0.2 to 0.4 and obtain Figure 6. We also assume that the ‘Residence Only’ withholding tax system is being used. We further assume that Tobin tax is charged when capital flows into a country’s market (capital inflows), and the rate is set at 1%. In
Figure 6, the vertical axis is the ratio of the local market weight when ‘a’ is 0.2 to the weight when ‘a’ is 0.4. With less unrealized capital gains (a=0.2), Tobin tax has a larger effect on rebalancing and therefore capital flows. But this difference is smaller when there is a large change in tax rate and therefore a large change in asset expected returns is made. So if asset expected returns change by a small amount, investors’ rebalancing will be more sensitive to Tobin tax in a volatile market (with small unrealized capital gains), and the Tobin tax will greatly reduce the trading volume between markets. In contrast, in a trending market (with large unrealized capital gains), investors will be more concerned about the current portfolio’s unrealized capital gains and their tax cost if redeemed, than the cost of Tobin tax. However, if asset expected returns change by a large amount, neither unrealized capital gains nor Tobin tax will be a major part of investors’ rebalancing considerations.

Finally, in Figure 7, a comparison is made between using a consistent Tobin tax rate globally (‘Tin=0.5%’ and ‘Tin&Tout=0.5%’) and using heterogeneous Tobin tax rates globally (‘Tin=0;0.5%;1%’ and ‘Tin&Tout=0;0.5%;1%’). We also assume that a ‘Residence Only’ withholding tax system is being used. The vertical axis is the ratio of the percentage change in a target market (Eurozone or U.K. market) weight to the percentage change in the U.S. market weight when the investment tax rate is cut or increased. First, we assume only capital inflows are subject to Tobin tax (the first chart of Figure 7). It shows that when the U.S. investment tax rate is cut, there will be capital

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18 Here means the Tobin tax rate in the Eurozone is 0, the Tobin tax rate in the U.K. is 0.005 and the Tobin tax rate in the U.S. is 0.01

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outflow from the Eurozone and the U.K. markets, and into the U.S. market. However, since only the inflow will be taxed, the capital flow will be subject to the U.S. Tobin tax only. The effective tax rate should be the same regardless of whether homogenous or heterogeneous Tobin tax rates are used globally. Thus, there will be no effect on the performance of markets.

In contrast, when the U.S. investment tax rate is increased, leading to a capital outflow from the U.S. market, heterogeneous Tobin tax rates will mean that all capital flowing into the U.K. market will be taxed at zero while capital flowing into the Eurozone market will be taxed at 1%. This difference in Tobin tax treatment will lead to increasingly more flow of capital into the U.K. market rather than into the Eurozone market, and consequently largely reduces the ability of the Eurozone market to attract overseas investment in the long-term. From the first chart of Figure 7, we can see that when the tax rate is increased, on average, 300% more capital will flow into the U.K. market when heterogeneous Tobin tax rates are used. Furthermore, if both outflows and inflows of capital are subject to Tobin tax, the result will be different (see the second chart of Figure 7). This difference occurs mainly when the U.S. investment tax rate is cut.

From the second chart in Figure 7, when the U.S. tax rate is cut, the rebalancing amount is different from that using a consistent Tobin tax rate globally. More capital will flow out of the U.K. market and less capital will flow out of the Eurozone market if heterogeneous investment tax rates are used globally. As a result, heterogeneous Tobin tax rates will lead to higher volatility and trading volume in a low Tobin-taxed
market (U.K) and lower volatility and trading volume in a highly Tobin-taxed market (Eurozone). This significantly reduces the ability of governments to intervene in the markets when necessary and reduces the ability to attract overseas investment.

6. Conclusion

This paper investigates the quantitative effects of investment taxes and Tobin tax on capital flows in global markets. We develop a post-tax portfolio optimization model with non-linear trading constraints and objective function. To undertake a quantitative examination of the influence of heterogeneous withholding and Tobin tax on global financial markets, we incorporate a broad range of the real-world trading constraints so investor behavior can be simulated more realistically. We quantify this influence by observing the rebalancing activities of rational investors under different tax settings.

In a comparison between residence- and source-based taxes on global investments, we find that the global optimal portfolio is highly sensitive to a change in regional investment tax rates. This sensitivity depends on the size of the tax rate change, market specifications, and the international investment tax environment (Residence Only, Source Only, or Mixed). In a uniform tax policy across countries, a source only tax union will, on average, have more capital transits in global markets than would be the case with a Residence Only tax union, and its optimal market portfolio will be more sensitive to regional tax policy changes. In a mixed tax system, Mixed (Double Taxation) between residence- and source-taxed markets will significantly reduce the attractiveness of the latter to investors, while the Mixed (Credit Method) will perform
much better (increasing the attraction of the market with a source-based tax by up to 20%). The experimental results also suggest that volatile markets, which are usually accompanied by low unrealized capital gains, are more sensitive to a government’s tax policy than trending markets.

Trading volume from rebalancing activities of rational investors (who seek to maximize the net Sharpe ratio) is highly sensitive to the implementation of a Tobin tax. This sensitivity varies with both market specifications and investment tax rules. A volatile market in a ‘Mixed (Credit Method)’ tax environment will be more sensitive to Tobin tax than a trending market in a ‘Mixed (Double Taxation)’ tax environment. Furthermore, our experiments show that the capital locking effects of Tobin tax is mainly dependent on its effective rate but not the taxation on the capital flow (taxing inflow only or outflow only), if a consistent Tobin tax rule is applied globally. When heterogeneous rules apply globally, for a market with relatively high Tobin tax rate, the inflow Tobin tax will have a much higher capital lock-out effect, and the outflow Tobin tax will have a much higher capital lock-in effect, in comparison to a globally consistent Tobin tax system. In other words, the capital locking effect of Tobin tax is enlarged significantly when heterogeneous Tobin tax rates are applied globally. As a result, it will be helpful if all countries reach an agreement on the implementation of Tobin tax. Otherwise, a relatively high Tobin tax will significantly reduce the appeal of local markets to foreign investors.
References


### Appendix

#### Table 1 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>((1, 1, \ldots, 1)')</td>
</tr>
<tr>
<td>(u'v)</td>
<td>(u_1v_1 + u_2v_2 + \ldots + u_nv_n) (inner product)</td>
</tr>
<tr>
<td>(u \circ v)</td>
<td>((u_1v_1, u_2v_2, \ldots, u_nv_n)') (Hadamard product)</td>
</tr>
<tr>
<td>(n_k)</td>
<td>number of investment assets in account (k)</td>
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<tr>
<td>(f_{kj})</td>
<td>percentage paid in management fees for the account of class (k) and country (j)</td>
</tr>
<tr>
<td>(\overline{d}_{kj})</td>
<td>expected dividends or income returns for the account of class (k) and country (j)</td>
</tr>
<tr>
<td>(\overline{g}_{kj})</td>
<td>expected capital gains for the account of class (k) and country (j)</td>
</tr>
<tr>
<td>(\ell_{kj}^{cg})</td>
<td>cumulative capital gains tax for the account of class (k) and country (j)</td>
</tr>
<tr>
<td>(\ell_{kj}^{cgm})</td>
<td>cumulative capital gains tax for the account of class (k) and country (j) for group (m)</td>
</tr>
<tr>
<td>(t_{kin}^{kj})</td>
<td>annual income tax for the account of class (k) and country (j) for group (m)</td>
</tr>
<tr>
<td>(t_{kin}^{kj})</td>
<td>annual income tax for the account of class (k) and country (j) for group (m)</td>
</tr>
<tr>
<td>(t_{j}^{in})</td>
<td>Tobin tax on capital inflows to the country (j)</td>
</tr>
<tr>
<td>(t_{j}^{out})</td>
<td>Tobin tax on capital outflows from the country (j)</td>
</tr>
<tr>
<td>(R_{jm}^{k0})</td>
<td>initial cumulative capital gains for the account of class (k) and country (j) for group (m)</td>
</tr>
<tr>
<td>(\tau_{jm}^{k0})</td>
<td>initial accumulated tax in the account of class (k) and country (j) for group (m)</td>
</tr>
<tr>
<td>(w_{jm}^{k0})</td>
<td>initial amount of wealth held in each asset for group (m)</td>
</tr>
<tr>
<td>(\Sigma)</td>
<td>covariance of assets</td>
</tr>
<tr>
<td>(C_{0jm})</td>
<td>External funding for local investment at the beginning of the period</td>
</tr>
<tr>
<td>(C_{0m})</td>
<td>external funding for overseas investment at the beginning of the period</td>
</tr>
<tr>
<td>(R_{jm}^{k1})</td>
<td>cumulative returns for the account of class (k) and country (j) after withdrawals</td>
</tr>
<tr>
<td>(\tau_{jm}^{k2})</td>
<td>final accumulated tax for the account of class (k) and country (j)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$TR_{kjm}$</td>
<td>net redemption value obtained from the account of class k and country j</td>
</tr>
<tr>
<td>$w_{k1}^{jm}$</td>
<td>amount of money held in each asset after rebalancing</td>
</tr>
<tr>
<td>$w_{k2}^{jm}$</td>
<td>final amount of money held in each asset</td>
</tr>
<tr>
<td>$l_k^{b}$</td>
<td>amount of money spent to buy an asset locally</td>
</tr>
<tr>
<td>$i_k^{b}$</td>
<td>amount of money spent to buy an asset internationally</td>
</tr>
<tr>
<td>$l_s^{kjm}$</td>
<td>amount of money obtained when selling an asset locally</td>
</tr>
<tr>
<td>$i_s^{kjm}$</td>
<td>amount of money obtained when selling an asset internationally</td>
</tr>
<tr>
<td>$x^2_{kjm}$</td>
<td>withdrawal from capital gains of assets for international transits</td>
</tr>
<tr>
<td>$x^3_{kjm}$</td>
<td>withdrawal taken from the initial capital international transits</td>
</tr>
<tr>
<td>$x^2_{kjm}$</td>
<td>withdrawal from capital gains of assets for local transits</td>
</tr>
<tr>
<td>$x^3_{kjm}$</td>
<td>withdrawal taken from the initial capital local transits</td>
</tr>
<tr>
<td>$\delta_{kjm}$</td>
<td>$\in [0,1]$, variable for assets</td>
</tr>
<tr>
<td>$C_{2m}$</td>
<td>income return at the end of the period</td>
</tr>
</tbody>
</table>
Figure 1 Residence-based vs. Source-based Investment Tax (a=0.2)

In Figure 1, the three charts show the comparison of four withholding tax systems without Tobin tax: Residence Only, Mixed (Credit Method), Mixed (Double Taxation) and Source Only. The title of each chart is the focused regional market. The horizontal axis is the investment tax rate (both income tax and capital gains tax) of the focused market and the vertical axis is the weight of the focused market in the obtained market portfolio.
Figure 2 Residence-based Investment Tax (a=0.2 or a=0.4)

In Figure 2, the two charts show the differential impact of Residence Only withholding tax on global markets with two distinct unrealized capital gains amount, ‘a’. The parameter ‘a’ is the assumed proportion of assets’ unrealized capital gains in each regional market. In the first chart, we assume only one regional market’s ‘a’ is changed from 0.2 to 0.4. In the second chart, we assume all three regional markets’ ‘a’ is changed from 0.2 to 0.4. The horizontal axis of these two charts are still the investment tax rate while the vertical axis is the ratio of the regional market’s weight with a=0.4 to its weight with a=0.2.
Figure 3 Tobin tax in residence-based only (a=0.2)

In Figure 3, the three charts show the comparison of different Tobin tax rules with a Residence Only withholding tax system. The title of each chart is the focused regional market. The horizontal axis is the investment tax rate while the vertical axis is the ratio of the focused market’s weight with Tobin tax to its weight without Tobin tax.
Figure 4 Tobin tax in credit method ($a=0.2$)
Figure 5 Tobin tax in source-based only (a=0.2)

Eurozone Market with Tobin Tax (Source)

UK Market with Tobin Tax (Source)

US Market with Tobin Tax (Source)
Figure 6 Tobin tax in residence-based only (a=0.2 or a=0.4)

Figure 6 shows the differential impact of Tobin tax on global markets with two distinct unrealized capital gains amount, ‘a’. The parameter ‘a’ is changed from 0.2 to 0.4 in all three regional markets. The horizontal axis is the investment tax rate while the vertical axis is the ratio of the focused regional market’s weight with a=0.2 to its weight with a=0.4.
Figure 7 Tobin tax in residence-based only (\(t=0.005\) vs \(t=0;0.005;0.01\))

Figure 7 shows the comparison of consistent and heterogeneous Tobin tax rules. In the consistent Tobin tax, all regional markets charge 0.5% on an international transit of wealth. In the heterogeneous Tobin tax, the U.K. market charges no Tobin tax, the U.S. market charges 0.5%, and the Eurozone market charges 1%. The first chart assumes only transits into a regional market will be charged a Tobin tax for that market while the second chart assumes both transits into or out of a regional market will be charged a Tobin tax for that market. The horizontal axis is the investment tax rate of the U.S. market while the vertical axis the ratio of the percentage change on the target market (the Eurozone or U.K. market) weight to the percentage change on U.S. market weight when investment tax rate is cut or increased.
Inflow&Outflow Tobin tax: all 0.005 v.s mixed

0.005 EU/US (US tax)
0.005 UK/US (US tax)
Mixed EU/US (US tax)
Mixed UK/US (US tax)