Stock market volatility in a simple framework

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Abstract
This paper provides a user-friendly framework to explain how variation in fundamental price-determining variables ‘translates into’ variation in the fundamental value of equities, based on the standard dividend-growth model. The framework is illustrated with UK data and with simple estimates of real interest rate forecasts and real dividend growth rate forecasts in the past. An important application of this approach is that stock market volatility can be analysed in terms of its component parts, unlike in previous research. Actual market volatility does not appear to be excessive when compared with the notional volatility implied by changes over time in our estimates of forecast real interest rates and forecast real dividend growth rates.

Keywords: Excess volatility; rational valuation; dividend-growth model; equity risk premium

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1. Introduction

The value of a share is determined, in theory, by the expected dividends and other cash payments which the share provides, and by the discount rate or rates at which the expected payments are discounted. The discount rate is given by the risk-free interest rate plus a risk premium. If the view of ‘the market’ changes about one or more of these price-determining variables, the share price should change. The main aim of this paper is to explain in a simple way how variation in the price-determining variables ‘translates into’ variation in fundamental value.

An important application of our analysis lies in the study of stock market volatility. It is widely believed that there are periods when actual stock market value deviates substantially from its fundamental value, resulting in increased market volatility. In other words, much of the observed market volatility is thought to be a result of ‘irrational’ valuations, ie valuations affected by behavioural factors that do not appear in standard finance models of asset value. The claim that prices are excessively volatile can be expressed as a claim that prices vary over time by more than is justified by variation in the price-determining variables. Our new framework will therefore help readers to form a better-grounded view as to whether markets are indeed excessively volatile.

The original debate on volatility was prompted by Shiller’s (1981) finding that stock market value varies over time much more than does the present value of subsequent dividends, assuming perfect foresight for dividends and interest rates. Campbell & Shiller (1988) allow forecasts of the real interest rate and real dividend growth rate to vary over time, and they use vector autoregression (VAR) to arrive at these forecasts. The equity premium is assumed to be constant. They conclude that changes in the forecasts are insufficient to explain the observed volatility of US market values. Shiller & Beltratti (1992) present similar evidence using UK data. This evidence raised the question as to whether the ‘excess’ volatility thus identified can reasonably be explained by changes in the expected equity premium, or whether the excess volatility is better seen as evidence for changes in investor sentiment that can not fully be accounted for by changes in the variables that affect value (Cochrane, 1991).

Subsequent research has questioned whether changes in forecasts of cash flows to equity and of discount rates are too small to explain the observed market volatility. Lettau &
Ludvigson (2005) find that ‘changing forecasts of stock market dividend growth do make an important contribution to fluctuations in the post-War U.S. stock market’ (p. 585), though their evidence is consistent with the existence of excess volatility. Ackert & Smith (1993) present evidence that, when share repurchases and cash payments resulting from acquisitions are added to dividends, the volatility of observed market values is not excessive, even assuming a constant discount rate. Larrain and Yogo (2008) compare the variation in dividend yield, in which the cash flows are given by dividends, and payout yield, in which cash flows are given by dividends plus repurchases less cash raised via share issues. Assuming an infinite future horizon, all of the variation in either measure of yield has to be explained by changes in forecast cash flows and discount rates. Using VAR forecasts, they find that 83% of the variation in dividend yield is explained by variation in the discount rate (their Table 9), consistent with excess volatility of market values. But, in contrast, 84% of the variation in payout yield is explained by variation in forecast payout growth, and changes in equity repurchase and issuance are ‘highly predictable’ (p. 220). So there is less support for the excess-volatility view, if yield is measured as payout yield instead of dividend yield. Chen & Zhao (2009) show that the variation in forecast discount rates is very sensitive to the specification of the VAR forecasting model. The conclusion about the relative importance of variation in discount rates or future cash flows in explaining the variation in equity returns is also very sensitive to the VAR specification.

For all the debate about the extent of excess volatility, the literature leaves readers with little understanding of how much variation in the forecast price-determining variables there has been, and how much variation is required to justify the observed volatility of market values. This is because most studies estimate the notional market values justified by price-determining variables from the Campbell-Shiller (1988) version of the dividend-growth model, or some variant thereof, combined with a VAR to estimate the year-by-year forecasts of the discount rates and cash flows. The variation over time in the forecasts of discount rates and cash flows, and their impact on the resulting estimate of rational market volatility, are opaque in such studies. The Campbell-Shiller (1988) framework is summarised in Appendix 1.

The current paper explores the market volatility justified by changes in price-determining variables in a much more accessible manner. In the interests of simplicity and transparency, our decomposition of the factors affecting value is based on the standard dividend-growth model. This means we assume that, at a given date, the values of the forecast discount rate and dividend growth rate are the same for each future year, and when the
forecast discount rate or growth rate changes, the change applies to the forecast rate for every future year. Our decomposition based on the dividend-growth model is new, as far as we know.

Our illustrations employ UK data for the period 1921-2008. An advantage of using UK data is that cash flows to investors are almost entirely in the form of dividends, even in recent years. We present base-case year-by-year forecasts of the real interest rate as at date $t$, $r_{real,t}$, and forecasts of the real growth rate of dividends as at date $t$, $g_{real,t}$, and show how altering the assumed variation in these forecasts affects the implied market volatility.

This approach to forecasting is similar to that of papers which estimate the expected equity premium in the past (Blanchard, 1993; Jagannathan et al, 2001; Arnott & Bernstein, 2002; Best & Byrne, 2002; Fama & French, 2002; Ilmanen, 2003; Claus & Thomas, 2003; Vivian, 2007). These papers infer the expected premium from some version of the dividend-growth model, using a variety of relatively simple methods to estimate the forecasts of investors, none of which involve a VAR. We believe that our base-case forecasts are reasonable, assuming that investors forecast a single future $r_{real,t}$ and $g_{real,t}$ at a given date $t$. Alternatively, we can assume that investors forecast different values for $r_{real,t+n}$ and $g_{real,t+n}$ for different future years $t+1$, $t+2$..., but that changes in these forecast values over time are equivalent to changes in single numbers for $r_{real,t}$ and $g_{real,t}$.

We first use the framework to estimate the expected equity premia in the past, and to estimate the contribution to market volatility of changes over time in the expected premium. We find that the mean of the expected premia is 3.3%, well below the mean of the actual premia, especially for the half-century 1950-99. This is consistent with previous research. We also find that, although there are substantial changes in the expected premium over time, the changes do not contribute to market volatility. This is because the changes in the expected premium often dampen down the changes in market value that would have occurred because of changes in $r_{real,t}$ and $g_{real,t}$.

We then infer the market values that would have arisen if the expected premia were fixed at their estimated mean. This enables us to compare the market volatility implied under a fixed expected premium with the actual volatility. Using our base-case forecasts of $r_{real,t}$ and $g_{real,t}$, with a fixed expected premium, volatility during 1921-2008 would have been somewhat greater than the actual volatility. That is, changes in the expected real interest rate and expected real dividend growth rate are sufficient to explain actual volatility, given our base-

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2 Another strand in the literature explores the forecasting of equity premia using lagged explanatory variables such as dividend yield (for example, Welch & Goyal, 2008).
case forecasts for these variables. Since these forecasts are reported explicitly in Section 3, readers are in a position to judge for themselves whether the forecasts appear to be excessively variable. We go on to reduce the volatility of the base-case forecasts of \( r_{\text{real},t} \) and \( g_{\text{real},t} \) to show the impact on the implied market volatility.

The rational forecasts that should determine market value at any given time cannot be known with certainty, whatever method of estimation is used.\(^3\) Our paper joins others, mentioned above, which argue that estimates of rational volatility are sensitive to the choice of forecasts.\(^4\) However, the main contribution is the transparent framework within which the forecasts of \( r_{\text{real},t} \) and \( g_{\text{real},t} \) determine fundamental value. It is hoped that the reader will be left with a much clearer grasp of the relationship between the volatilities of the forecasts and the resulting notional market volatility.

2. Decomposition of equity and bond returns

This section offers a new and straightforward analysis of the equity and bond returns for a given year in terms of changes in the variables that should affect equity and bond values.

2.1 Equity

The expected nominal equity return as at date \( t, r_t \), is estimated by

\[
r_t = D_t (1 + g_t)/P_t + g_t
\]

where \( D_t \) is the total dividends paid by listed companies in a given index during the year ending at date \( t \), \( P_t \) is the total market capitalisation of the ordinary shares in the index at date \( t \), and \( g_t \) is the expected value of the annual growth rate of dividends, as at date \( t \). Our notation uses capitals for actual observable values, and small letters for expected values.

Return due to unexpected dividend growth in first year

First we isolate the effect on the equity return of unexpected dividend growth during the year, that is, the notional return assuming no change during the year in \( r_t \) or \( g_t \), less the expected return. It follows from equation (1) that the prospective dividend yield remains the same from one year to the next, in which case

\(^3\) The standard test for whether a given forecast is rational is to examine whether the forecast errors are related to information known at the time the forecast was made. But the relevant horizon for the real interest rate and real dividend growth rate is infinity, so it is uncertain over what interval to measure the forecast errors.

\(^4\) Estimates of the mean expected premium inferred from the dividend-growth model are more robust. The conclusion that the mean observed premium in the UK or USA was higher on average than the mean expected premium during the second half of the twentieth century is not sensitive to the specific forecasts chosen or methodology used.
\[
D_t(1 + g_t)/P_t = D_t(1 + G_t)(1 + g_{t+1})/P_{t+1}
\]
and with \( g_t = g_{t+1} \),
\[
D_t/P_t = D_t(1 + G_t)/P_{t+1}
\]
where \( G_t \) is the actual nominal dividend growth rate for the year starting at date \( t \). Thus, to maintain the same yield at \( t+1 \) as at \( t \), we must have
\[
P_{t+1} = P_t(1 + G_t)
\]
(2)
So the actual equity return with \( r_t \) and \( g_t \) fixed, \( R(rgfixed)_t \), is
\[
R(rgfixed)_t = (D_{t+1} + P_{t+1})/P_t - 1
= \{D_t(1+G_t) + P_t(1+G_t-P_t)/P_t
\]
= \( D_t(1 + G_t)/P_t + G_t \)
= \( G_t(1 + D_t/P_t) + D_t/P_t \)
of which the expected return is given by
\[
r_t = g_t(1 + D_t/P_t) + D_t/P_t
\]
(3), from re-arranging (1)
and the effect on return of unexpected dividend growth during the year starting at date \( t \), \( R(\Delta div)_t \), is given by
\[
R(\Delta div)_t = R(rgfixed)_t - r_t
= (G_t - g_t)(1 + D_t/P_t)
\]
(4)

Return due to changes in expected return or expected dividend growth

We now consider the return that is due to a change in \( r_t \) or \( g_t \).
Re-arranging (1) again,
\[
(r_t - g_t)/(1 + g_t) = D_t/P_t
\]
(5)
So a change in \( r_t \) or \( g_t \), implies a change in yield, and the change in yield can be written as:
\[
\Delta[(r-g)/(1 + g)]_t = (r_{t+1} - g_{t+1})/(1 + g_{t+1}) - (r_t - g_t)/(1 + g_t)
\]
We know from (2) that the price at \( t+1 \) with no change in yield must be \( P_t(1 + G_t) \). So the return due to a change in yield, or in \( r_t \) or \( g_t \), \( R\{\Delta[(r-g)/(1 + g)]\}_t \), can be written as
\[
R\{\Delta[(r-g)/(1 + g)]\}_t = [P_{t+1} - P_t(1 + G_t)]/P_t
\]
It follows that (see Appendix 2 for proof):
\[
R\{\Delta[(r-g)/(1 + g)]\}_t = \frac{[D_t/P_t - D_{t+1}/P_{t+1}](1+G_t)}{D_{t+1}/P_{t+1}}
\]
(6)
The return due to a change in \( r_t \) or \( g_t \), can also be analysed as the sum of the returns due to changes in the components of \( r_t \) and \( g_t \). We have
\[
r_t = r_{Freal,t} + i_t + irp_t + erp_t
\]
(7)
and

\[ g_t = g_{\text{real},t} + i_t \]

That is, the nominal expected return on equity is given by the sum of the expected real interest rate, \( r_{\text{Freal},t} \), the expected rate of inflation, \( i_t \), the inflation risk premium, \( \text{irp}_t \), and the expected equity risk premium, \( \text{erp}_t \). The expected nominal growth rate of dividends is the sum of the expected real growth rate of dividends, \( g_{\text{real},t} \), and the expected rate of inflation. We use the approximate, additive, formula for nominal rates for simplicity.

Given the components of \( r_t \) and \( g_t \), we can write

\[
\frac{(r_t - g_t)}{(1 + g_t)} = \frac{[r_{\text{Freal},t} + i_t + \text{irp}_t + \text{erp}_t - (g_{\text{real},t} + i_t)]}{(1 + g_t)}
\]

Now we can use (5), (6) and (8) to express the return due to a change in \( r_t \) or \( g_t \) as the sum of the returns due to changes in the components of \( r_t \) and \( g_t \):

\[
R\{\Delta[(r - g)/(1 + g)]\}_t = R(\Delta r_{\text{Freal}})_t + R(\Delta \text{irp})_t + R(\Delta \text{erp})_t - R(\Delta g_{\text{real}})_t
\]

where

\[
\frac{R(\Delta r_{\text{Freal}})_t}{(1 + g_t)} = \frac{[r_{\text{Freal},t}/(1 + g_t) - r_{\text{Freal},t+1}/(1 + g_{t+1})](1 + G_t)}{(r_t - g_{t+1})/(1 + g_{t+1})}
\]

using equation (6), and analogously for the other variables. The denominator of (10) follows by substituting (5) into the denominator of (6); the numerator follows from (5) and (6), and from the fact that \( r_{\text{Freal},t} \) is a component of \( r_t \).

**Equity risk premium**

All the variables are estimated directly except the expected equity risk premium, \( \text{erp}_t \). This is calculated as the residual expected return: it is the difference between the expected equity return at date \( t \) and the expected return on the risk-free asset, \( r_{F,t} \):

\[
\text{erp}_t = r_t - r_{F,t} = r_t - (r_{\text{Freal},t} + i_t + \text{irp}_t)
\]

\( r_t \) is estimated via the re-arranged dividend-growth formula in (1), and \( r_{F,t} \) is proxied by the yield on undated government bonds. Our methods of estimating \( r_{\text{Freal},t} \), \( i_t \) and \( \text{irp}_t \), explained below, ensure that these components sum to give \( r_{F,t} \). Because \( \text{erp}_t \) is calculated as the residual expected return, our decomposition of the return for a given year is exact: the component returns on the right hand side of (9), which include \( R(\Delta \text{erp})_t \), sum exactly to give the return due to a change in dividend yield in (6), and we have

Actual return, \( R_t = \) Expected return, \( r_t + \\
Return due to unexpected dividend growth, \( R(\Delta \text{div})_t + \\
Return due to a change in dividend yield, \( R\{\Delta[(r - g)/(1 + g)]\}_t \\

Note that a change in the expected rate of inflation, $i_t$, does not affect the notional return on equity due to a change in $r_t$ or $g_t$. This is because $i_t$ is included additively in both $r_t$ and $g_t$, and therefore it exactly cancels out. The inflation risk premium, however, is a component of the discount rate but not of the expected rate of dividend growth, and so a change in its value affects the notional return for the relevant year. Another point to notice is that the absolute size of the return arising from a given percentage change in one of the variables is negatively related to the dividend yield at date $t+1$. That is, the impact on return of a given percentage change in one of the variables is greater when the yield is low.\(^5\)

2.2 Bonds

The return on bonds can be decomposed in a similar manner. We proxy $r_{F,t}$ by the yield on 2.5% consols (undated government bonds). With undated bonds, the price is given by

$$P_t = \frac{Y}{r_{F,t}} \quad (11)$$

where $Y$ is the annual interest payment, which is fixed in nominal terms. A change in price is caused by a change in the discount rate. It follows from (11) that if $r_{F,t}$ is expected to be constant at date $t$, the return that is due to a change in the discount rate, $R_F(\Delta r_F)_t$, is given by

$$R_F(\Delta r_F)_t = \frac{P_{t+1}}{P_t} - 1 = \frac{(r_{F,t} - r_{F,t+1})}{r_{F,t+1}}$$

The discount rate is given by the real interest rate plus the expected rate of inflation plus the inflation risk premium. So the return due to a change in the discount rate can be explained by changes in these variables:

$$R_F(\Delta r_F)_t = R_F(\Delta r_{Freal})_t + R_F(\Delta i)_t + R_F(\Delta irp)_t \quad (12)$$

where $R_F(\Delta r_{Freal})_t = (r_{Freal,t} - r_{Freal,t+1})/r_{F,t+1}$, and analogously for $i_{t,t+1}$ and $irp_{t,t+1}$.

3. Estimation of expected dividend growth rates and real interest rates in the past

We use UK data, and our sample period is 1921-2008. Requisite data are available for 1900 onwards, but we start in 1921 to avoid the period during and shortly after the 1914-18 war, when dividends were extremely volatile, and to ensure that we have at least 15 years of past dividend growth. The data are mostly from the Equity-Gilt Study, produced annually by Barclays Capital, and from Datastream. Details of the data sources and of our returns

\(^5\) Small changes in forecasts that affect all future periods can have a surprisingly large impact on the volatility of the implied market values, especially at low yields. For example, suppose that $r_t = 5\%$ and $g_t = 3\%$, and so the yield at date $t$ is $(5\% - 3\%)/(1.03) = 1.94\%$. A 10\% fall in $r_t$ gives a yield at date $t+1$ of 1.46\%, which implies a capital gain of 32.9\%. 

calculations are shown in Table 1. The forecasts and their volatilities are intended to be easy to understand, as well as plausible.

Table 1 around here

*Expected real dividend growth rate*

A number of variables and combinations of variables were assembled and analysed for their ability to reflect long-term real growth expectations, utilising the fact that a negative correlation between the equity market yield and such expectations should be anticipated. The following proved to be the most statistically significant historically:

\[
g_{\text{real},t} = \overline{G_{\text{real}}(t-15, t+9)}
\]  \hspace{1cm} (13)

where \( G_{\text{real}}(t-15, t+9) \) is the geometric mean real growth rate of dividends during the 25-year period that starts at date \( t-15 \) and ends at date \( t+9 \). The real growth for a single year is

\[
G_{\text{real},t} = \frac{(1 + G_t)(1 + I_t) - 1}{(1 + I_t)}
\]

where \( G_t \) is the percentage growth in dividends for year \( t \) and \( I_t \) is inflation. The estimate in (13) is a moving average measured over a long period, and so it is normally slow-moving. The combination of 15 years of past growth data, known as at date \( t \), and ten years of forward data, unknown at date \( t \), is meant to capture the idea that investors base their expectations primarily on observed growth in recent years, but with some ability to adjust for changing economic circumstances. The estimates of \( g_{\text{real},t} \) in Arnott & Bernstein (2002) and Ilmanen (2003) are also moving averages. We ignore repurchases and other cash payments to shareholders.

The above method results in estimates of negative expected real growth for several years in the 1920s, 1940s and 1970s. We take the view that an expectation of negative real growth for ever is implausible, and we therefore set a minimum expected real growth rate of 0% pa. The estimates are below 3.0% pa except for one outlier of 4.3% pa in 1935. We cap this estimate at 3.0% pa. These constraints reduce the variation in \( g_{\text{real},t} \), which reduces the variation in returns that can be ‘explained’ by changes in this variable. The estimates we use

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\(^6\) For 2000 onwards there are fewer than ten future years between date \( t \) and the end of 2008. For these years we use the growth rates for as many future years as there are available.

\(^7\) Annual repurchases in the UK were below 0.1% of market value until 1995, and had risen to 0.9% by 2004 (Vivian, 2007). Our results regarding volatility are almost the same if we add Vivian’s estimated repurchases to dividends.
are shown in Figure 1. The mean of $g_{\text{real},t}$ is 0.99%, the standard deviation is 0.81%, and the mean of the absolute values of the changes is 0.32%.

Figure 1 around here

*Expected real interest rate*

For the period 1982-2008 the expected real rate of interest, $r_{\text{Freal},t}$, is given by the yield on 20-year index-linked government bonds, which were introduced in 1981. The index-linked yield provides a good estimate of the expected long-term real interest rate at a given time, although the market was somewhat illiquid in its early years.

The expected rate of inflation, $i_t$, is estimated for 1982-2008 as follows. We define the inflation gap, $I_{\text{gap}}_t$, as the difference between the nominal interest rate and the expected real interest rate on an asset with zero default risk. The inflation gap for long bonds is estimated by the difference between the nominal yield on 2.5% Consols, $r_{F,t}$, and the yield on 20-year index-linked gilts. The expected rate of inflation is given by the estimated inflation gap less an estimated premium for inflation risk, $i_{\text{rp}}$. The inflation risk premium is given simply by

$$irp_{(1982-08)} = 0.2I_{\text{gap}}_t$$

This assumes that there is a positive relation between the inflation risk premium and expected inflation, and there is evidence that such a relation exists (Breedon & Chadha, 2003). Evidence to justify an inflation risk premium of up to 20% of the inflation gap is in Shen (1998) and Garcia & Werner (2010). The expected rate of inflation is estimated by

$$i_{t}(1982-08) = 0.8I_{\text{gap}}_t$$

We shall also run the calculations with no inflation risk premium.

For the period 1921-81, we estimate expected inflation from the yield on consols. A regression of the yield $Y_t$ on future inflation shows that the yield provides quite a good forecast, at least for inflation during the subsequent decade:

$$\bar{I}_{t, t+9} = 0.323\% + 0.616Y_t$$

$$0.40 \quad 5.69 \quad R^2 = 0.30$$

where $\bar{I}_{t, t+9}$ is the geometric mean rate of inflation for the ten years $t$ to $t+9$ and the numbers are OLS regression coefficients with $t$-statistics beneath. So our estimate of expected inflation for 1921-81 is

$$i_{t}(1921-81) = 0.323 + 0.616Y_t$$

(14)

The expected real interest rate for 1920-81 is then given by
\[ r_{\text{Real},t}(1921-81) = r_{F,t} - (i_t + irp_t) \]

where \( r_{F,t} \) is the prevailing yield on 2.5% Consols, as above, and

\[ irp_t(1921-81) = 0.2i_t(1921-81)/0.8 \]

so that \( i_t(1921-81) = 0.8gap_t \), as for \( i_t(1982-08) \). For both periods, \( r_{\text{Real},t} \) is exactly equal to \( r_{\text{Real},t} + i_t + irp_t \), as mentioned in Section 2.1.

The estimates of the expected real interest rate are shown in Figure 2.

The rate varies between 0.2% and 4.4% pa. As this seems a reasonable range within which the expected rate could vary, we make no adjustments. The mean of \( r_{\text{real},t} \) is 1.56%, the standard deviation is 1.26%, and the mean of the absolute values of the changes is 0.19%. If it is accepted that the yield on index-linked gilts provides a fairly precise estimate of the expected real interest rate at a given time, the evidence from the index-linked market shows that the expected real rate does vary year-by-year. In fact the mean of the absolute values of the changes is 0.30% for 1982-08 from the index-linked market, compared with 0.15% for 1921-81 from our alternative estimates.

4. Results

In this section, we illustrate our approach for the period 1921-2008, first with an inferred (variable) expected equity premium and then with a fixed expected equity premium. We then calculate simulated volatilities under a range of alternative assumptions. In particular, we vary our estimates of standard deviations of the main real variables, assess the implications for the standard deviation of fixed-premium returns, and make comparisons with the standard deviation of actual returns.

4.1 Results with variable expected equity premium

Table 2 shows the results for the case in which the expected equity premium is inferred and varies from year to year, as described above. We first briefly discuss the results regarding the equity premium. The arithmetic mean expected premium for 1921-2008 is 3.3%, whereas the ex post premium is 4.9%. If we take the 50 years 1950-99, the mean expected premium is 3.0%, compared with an ex post premium of 9.1% (not shown in the table). Thus the results agree with those of other UK and US studies, cited in the Introduction, that estimate the expected premium in the past and find that the historic equity premium in the
second half of the twentieth century provides an upwardly biased estimate of the expected premium.

Table 2 around here

The difference of 1.6 percentage points between the actual and expected risk premium for 1921-2008 is explained by an actual mean return on equity that is 2.9 percentage points higher than the expected return, partly offset by an unexpected positive return on government bonds of 1.2%. According to our decomposition, the reasons for the higher-than-expected ex post premium are a decline in the expected equity premium (+1.2 percentage point contribution to the ex post premium) and higher real growth of dividends than expected (+1.0 point). These factors that increased the observed premium are partly offset by changes in expected inflation that increased the return on bonds (−0.4 point contribution to the premium). The inference of a declining expected premium echoes that of previous studies, especially Fama & French (2002). But we also find that unexpectedly high dividend growth is an important contributor to the unexpectedly high ex post premium.

We now turn to the results relating to volatility, starting with the impact of changes in the expected equity premium. The expected premium for each year has a mean of 3.3%, a standard deviation of 1.8%, and the mean of the absolute values of the changes is 0.8%. So it exhibits substantial variation over time. The standard deviation of the returns due to changes in the expected equity premium is 22.7%, which is close to the standard deviation of 23.9% for the actual returns. However, the changes in the expected premium contribute nothing to the volatility of the observed returns, because they often dampen down the return that would have arisen had the premium not changed. One way of showing this is in Figure 3. There are two bars for each year. The first bar is the actual return on equity less the return arising from the change in the expected premium: \( R_t - R(\Delta{erp})_t \). This difference is the return for the year that would have arisen, had the expected premium not changed from its value in the previous year (it is not the same as the return assuming a constant expected premium every year). The second bar shows the return that is ascribed to the change in the expected premium, \( R(\Delta{erp})_t \). The two returns (bars) combined give the actual return, \( R_t \). For clarity, Figure 3 shows these results for 1960-2008 only.

Figure 3 around here
Figure 3 shows that for the majority of years (61% during 1960-2008; 65% during the full sample period), the return due to the change in expected premium has the opposite sign from the return due to the other variables. Consistent with the visual impression, the correlation coefficient for the series $R_t - R(\Delta \text{erp})$, and $R(\Delta \text{erp})$, is $-0.32$. During the four years 1995-98, for example, the equity returns would have been even higher had the expected premium not been rising. The major exceptions are the extreme years of 1974 and 1975, when the returns due to changes in the expected premia greatly augmented the returns ascribed to other factors.

The reader might feel that a mean of the changes in the inferred premium of 0.8% implies implausibly large jumps from one year to the next. The consensus in other research is that, while the expected premium probably does change over time, it does so gradually. However, our estimates of the changes in expected premia should not be taken too literally. $\Delta \text{erp}_t$ is calculated as a residual: it is the change in the expected equity return (discount rate) that must have arisen given the actual equity return for the year starting at date $t$, and given the changes in the other variables that determine the notional return with no change in the premium. One possible explanation for the variation in the expected premium is that our estimates of $r_{\text{real},t}$ and $g_{\text{real},t}$ are not variable enough: investor expectations regarding these variables change through time by more than we have estimated, causing the inferred premium to overstate the actual changes in the expected premium. For example, during the extreme year of 1975, the expected premium falls from 6.5% at the start of 1975 to 0.6% at the start of 1976. The estimated real interest rate falls from 3.7% to 3.0%. There is no change in the estimated forecast real dividend growth rate, which is zero for both years. The actual equity premium for 1975 is +111.0%, most of which is ‘explained’ in our analysis by a fall in the expected premium. But if the fall in the expected real interest rate was greater than we measure, or if investors became more optimistic about real dividend growth during 1975, our estimate of the change in the expected premium is exaggerated.

Alternatively, for some years the return we ascribe to a change in the expected premium could be viewed as having an irrational component, and the apparent changes in the expected premium that we measure would then be seen to be a symptom of irrational pricing, as discussed in the Introduction. Whatever the interpretation of the changes in the expected premium, our findings to this point suggest that the observed volatility for the sample period can be explained without assuming that there were changes in the expected premium. We now show this more directly.
4.2 Results with fixed expected premium

This section presents the volatility that would have arisen had the expected premium been fixed. We set the expected premium at 3.3% every year, which is the arithmetic mean over the sample period that we have already inferred. The values of $r_{Freal,t}$, $i_t$, $irp_t$ and $g_{real,t}$ are unchanged. Using a similar approach to that in Section 2.1, the simulated expected return with a fixed premium, $Simr_t$, and the return due to unexpected dividend growth, $R(Sim\Delta div)_t$, are calculated as follows:

$$Simr_t = r_{Freal,t} + i_t + irp_t + 3.3\%$$  \hspace{1cm} \text{compare with (7)}

$$= g_t[1 + Sim(D/P)_t] + Sim(D/P)_t$$  \hspace{1cm} \text{(15), compare with (3)}

$$R(Sim\Delta div)_t = (G_t - g_t)[1 + Sim(D/P)_t]$$  \hspace{1cm} \text{(16), compare with (4)}

where the simulated dividend yield is given by

$$Sim(D/P)_t = (Simr_t - g_t)/(1 + g_t)$$  \hspace{1cm} \text{compare with (5)}

The simulated return due to a change in $Simr_t$ or $g_t$ is the sum of the returns resulting from changes in the expected real interest rate, the inflation risk premium and the expected real growth rate of dividends:

$$R\{\Delta[(Simr - g)/(1 + g)]\}_t$$  \hspace{1cm} \text{compare with (7)}

$$= SimR(\Delta r_{Freal})_t + SimR(\Delta irp)_t - SimR(\Delta g_{real})_t + \text{balancing term}$$  \hspace{1cm} \text{(17)}

where

$$SimR(\Delta r_{Freal})_t = [r_{Freal,t} (1 + g_t) - r_{Freal,t+1}(1 + g_{t+1})] \times (1 + G_t)$$  \hspace{1cm} \text{(18)}

$$\div (Simr_{t+1} - g_{t+1})(1 + g_{t+1})$$

and $SimR(\Delta irp)_t$ and $SimR(\Delta g_{real})_t$ are defined analogously. Although the expected premium is fixed at 3.3%, and so there is no return due to a change in the premium, a small balancing term is needed, defined as

$$\text{balancing term} = [3.3\%/(1 + g_t) - 3.3\%/(1 + g_{t+1})] \times (1 + G_t)$$

$$\div [(Simr_{t+1} - g_{t+1})(1 + g_{t+1})]$$

The total simulated return is the sum of (15), (16) and (17):

$$SimR_t = Simr_t + R(Sim\Delta div)_t + R\{\Delta[(Simr - g)/(1 + g)]\}_t$$

The results are shown in Table 3. The key finding is that the simulated returns with the expected premium fixed at 3.3% are at least as volatile as the actual returns.\(^8\) The standard

\(^8\) Although the expected premium is fixed at its sample mean, the simulated returns differ from the actual returns, and their means differ. This is because the impact on returns of changes in $Simr_t$, which incorporates the fixed premium, differs from the impact of changes in $r_n$ which incorporates a variable premium. It is the presence of $Simr_{t+1}$ in the denominator of (18) that causes the simulated returns arising from changes in $r_{Freal,t}$ for example, to differ from the unsimulated returns, given by (10).
deviation of the fixed-premium returns is 28.1%, compared with the standard deviation of the actual returns of 23.9%. The standard deviation of the fixed-premium returns would have to be below 16.5% for the fixed-premium returns to be significantly less volatile than the actual returns at the 1% level, using a one-tailed $F$-test on the ratio of the variances. Table 3 also shows that changes in $g_{real,t}$ are the most important cause of variation in the fixed-premium returns in the full sample.

Table 3 around here

We examine the period 1982-2008 separately (but we do not report the results in detail). Our estimates of the expected real interest rate are more reliable for this period, and the expected real dividend growth rate is less variable than in earlier years; the mean of the absolute values of $\Delta g_{real,t}$ is 0.24 for 1982-08, compared with 0.35 for 1921-81. The average expected premium for 1982-08 is 1.6%, so 1.6% is the fixed premium used to calculate the fixed-premium returns. The standard deviation of the actual returns on equity during 1982-2008 is 17.0%; the standard deviation of the fixed-premium returns is 21.0%. So, as for the full sample, the fixed-premium returns are at least as volatile as the actual returns. For 1982-2008 changes in the expected real interest rate are the most important source of variation in the fixed-premium returns.

We also observe that the fixed-premium returns are related to the actual returns. In 70% of the years in the full sample the two returns have the same sign, and the correlation coefficient for the two series is 0.24 ($t = 2.25$). These results indicate that the fundamentals that are supposed to affect value in the fixed-premium dividend-discount model do have significant explanatory power. Figure 4 shows the two returns for each year, for 1960-2008.

Figure 4 around here

### 4.3 Simulated volatilities under alternative assumptions

First we assume that there is no inflation risk premium. This assumption is made in a number of previous studies such as Blanchard (1993) and Ilmanen (2003). For 1921-81 we estimate the expected real interest rate by subtracting our estimate of expected inflation, with no added premium, from the consols yield. For 1982-08 we assume that the entire inflation gap measured via equation (14) represents expected inflation. These adjustments result in a standard deviation of the fixed-premium returns of 22.9%, less than the 28.1% of the base-
case fixed-premium returns, but little different from the standard deviation of the actual returns of 23.9%.

An advantage of our method is that the forecasts that determine the predicted changes in equity values are explicit. For predicted volatility with a fixed premium to be less than actual volatility, the estimated values of \( r_{Fr\text{eal},t} \) and \( g_{\text{real},t} \) would have to be less variable than they are in Figures 1 and 2. Table 4 shows the volatilities of the fixed-premium returns, with an inflation risk premium, under various assumptions about the volatilities of \( r_{Fr\text{eal},t} \) and \( g_{\text{real},t} \). It shows directly how changing the volatilities of the price-determining variables changes the volatility of the resulting notional market returns. Each cell reports the standard deviation of the fixed-premium returns resulting from applying differing values of \( x \) in the following formula

\[
\Delta r_{Fr\text{eal},t}^* = \text{av}(r_{Fr\text{eal},t}) + x[\Delta r_{Fr\text{eal},t} - \text{av}(r_{Fr\text{eal},t})]
\]

where \( \Delta r_{Fr\text{eal},t}^* \) is the adjusted change in the expected real interest rate for year \( t \) used to calculate the fixed-premium returns, \( \text{av}(r_{Fr\text{eal},t}) \) is the arithmetic mean of \( r_{Fr\text{eal},t} \) for the sample period, and we select a value for \( x \) between 1.0 and 0.0. \( x = 1 \) means that the year-by-year changes in \( r_{Fr\text{eal},t} \) are unaltered; \( x = 0 \) means that there is no variation in \( r_{Fr\text{eal},t} \). The same formula is used to vary the volatility of \( g_{\text{real},t} \). The formula results in smaller year-by-year changes in the relevant variable, while preserving its mean value.

Table 4 around here

Table 4 shows the combinations of adjustments needed for the standard deviation of the fixed-premium returns to be less than 16.5%, ie significantly less than the standard deviation of the actual returns. For example, if the volatility of \( g_{\text{real},t} \) were 0.8 times its actual level, the volatility of \( r_{Fr\text{eal},t} \) would need to drop to 0.4 times its actual level for the fixed-premium volatility to fall below 16.5%. If both \( g_{\text{real},t} \) and \( r_{Fr\text{eal},t} \) were constant, the fixed-premium volatility would be 8.9%. This is the volatility of the returns with a constant yield, most of which is volatility we attribute to ‘unexpected’ year-by-year changes in actual dividends paid (see equation 4). Table 4 also shows that the fixed-premium volatility is more sensitive to the volatility of \( g_{\text{real},t} \) than of \( r_{Fr\text{eal},t} \). This arises because the year-by-year changes are larger for \( g_{\text{real},t} \) than for \( r_{Fr\text{eal},t} \) in the full sample. Of course, if the base-case variation in \( g_{\text{real},t} \) or \( r_{Fr\text{eal},t} \) were felt to be too low, \( x \) in equation (19) would exceed one and the fixed-premium volatility would be higher than in the base case.
5. Conclusion

We present a transparent framework for analysing the impact on fundamental values of year-by-year changes in estimates of the expected real interest rate and the expected real growth rate of dividends. Our decomposition of market returns, based on the traditional dividend-growth model, has not been derived before. An important application of our analysis is that it helps the reader to appreciate the relationships between the volatility of price-determining variables and the volatility of fundamental values. We can see how variable over time the expected real interest rate and real dividend growth rate need to be in order for the observed market volatility to be justified. It is impossible to gain an understanding of this from previous research, which employs the Campbell-Shiller logarithmic version of the dividend-growth model combined with forecasts of the real interest rate and real dividend growth rate derived from vector autoregression. Our analysis also enables the equity premia expected in the past to be inferred.

We believe that our estimates of forecasts of $r_{real,t}$ and $g_{real,t}$ in the past are reasonable. They are derived from simple methods, of the type used by authors who have inferred the equity premium expected in the past. The average of the estimated expected premium during the sample period is 3.3%, which is in line with the estimates in previous studies. We find that changes in our forecasts of the expected real interest rate and expected growth rate of dividends are sufficient to explain the observed volatility of the UK stock market during 1921-2008. This is the case whether we allow the expected equity premium to vary, or whether we estimate the market returns which would have arisen had the expected premium been fixed. Readers can, literally, see what the forecasts look like (Figures 1 and 2) that produce our results, and judge for themselves whether the forecasts are too volatile. The standard deviation of both the forecasts of $r_{real,t}$ and $g_{real,t}$ would have to drop by about one third for the notional market volatility with a fixed premium to be the same as actual market volatility (Table 4).

Any forecasting method that produces year-by-year variation in $g_{real,t}$ and $r_{Freal,t}$ that is similar to the variation in our estimates is likely to give similar results regarding volatility. For example, a study that has similar implications to ours regarding volatility is Blanchard (1993). His estimates of $r_{Freal,t}$ and $g_{real,t}$ appear to fluctuate year-by-year at least as much as our estimates, although the range of Blanchard’s estimates is somewhat greater than ours for the expected real interest rate, and somewhat less for the expected real growth rate. In the light of our results for the UK, it is almost certain that Blanchard’s estimates of $r_{Freal,t}$ and $g_{real,t}$ would be more than sufficient to explain the observed US market volatility.
Using the framework in this paper, readers can readily calculate the expected equity premium, or the notional volatility of market returns, by inserting their own year-by-year forecasts of $r_{real,t}$ and $g_{real,t}$. 
Appendix 1: Measuring Rational Volatility - The Campbell-Shiller Framework

Campbell & Shiller (1988) show that the dividend yield justified by fundamentals at date $t$ is given approximately by:

$$\text{Est}[\ln(D_t/P_t)] = \text{Est}(\ln D_t - \ln P_t) \approx \sum_{n=0}^{\infty} \rho^n(\ln r_{\text{real},t+n} - \ln g_{\text{real},t+n})$$

where $\text{Est}$ stands for estimated, $\rho$ is a constant discount factor, $r_{\text{real},t+n}$ is the expected real one-year discount rate and $g_{\text{real},t+n}$ is the real dividend growth forecast at date $t$ for the $n$th year after the year starting at $t$.

A VAR produces forecast values of $(\ln r_{\text{real},t+n} - \ln g_{\text{real},t+n})$ that are different for each future year $t+1$, $t+2...$. Hence, the model in (20) is admirably suited to being estimated by means of a VAR. As we go from date $t$ to $t+1$, the forecasts will typically change. The vector of variables being forecast for a given future year ending at date $t+n$, $z_{t+n}$, is given by

$$z_{t+n} = A^n z_t$$

where $A$ is the matrix of coefficients estimated on the variables by the VAR, and $z_t$ is the vector of the values of the variables observed as at date $t$. Lagged values of variables can also be included. In Campbell & Shiller (1988), the variables used to forecast $(\ln r_{\text{real},t+n} - \ln g_{\text{real},t+n})$ are $(\ln D_t - \ln P_t)$ and $(\ln R_{\text{real},t-1} - \ln G_{\text{real},t-1})$, and these variables lagged one period, where $R_{\text{real},t-1}$ and $G_{\text{real},t-1}$ are, respectively, the observed real discount rate and dividend growth rate for the year ending at date $t$. Let $v$ be the vector that picks out the element $(\ln R_{\text{real},t-1} - \ln G_{\text{real},t-1})$ in $z_t$: $v' z_t = \ln R_{\text{real},t-1} - \ln G_{\text{real},t-1}$. Then we can write

$$\text{Est}(\ln D_t - \ln P_t) \approx \sum_{n=0}^{\infty} \rho^n(\ln r_{\text{real},t+n} - \ln g_{\text{real},t+n}) = \sum_{n=0}^{\infty} \rho^n v' A^{n+1} z_t = v' A z_t (I - \rho A)^{-1}$$

where $I$ is the identity matrix. The last expression uses the formula for the sum of a geometric series; this step requires that all the coefficients in $A$ be less than one. It means that, for the VAR forecasts to be usable, they imply the following: for a given change in the forecast for the next period, i.e. for a given change in $(\ln r_{\text{real},t} - \ln g_{\text{real},t})$, the change in the forecast for each subsequent period diminishes. This differs from the assumption in our paper that when the forecast discount rate or growth rate changes, the change applies to the forecast for every future year.

---

9 The model includes a constant term which we have omitted for clarity. The constant does not affect their estimates of rational volatility. $\rho$ is given in Campbell & Shiller (1988) by the sample mean of $P_t/(P_t + D_t)$, which is 0.937. Their results are not sensitive to alternative values for $\rho$ between 0.900 and 0.975.
The coefficients of the VAR are not always reported. But even if they are reported, as for example in Campbell & Ammer (1993), it is impossible to gain any impression of the variation in the estimated forecasts of $r_{\text{real}_{t+n}}$ and $g_{\text{real}_{t+n}}$, and of how the variation in the forecasts affects the estimated rational volatility of market values.
Appendix 2: Proof of Equation (6)

\[ R \{ \Delta[(r - g)/(1 + g)] \}_t = [P_{t+1} - P_t(1 + G_t)] \div P_t \]

\[ = \left[ \frac{P_{t+1}}{(1 + G_t)P_t} - 1 \right] (1 + G_t) \]

\[ = \left[ \frac{P_{t+1}}{(1 + G_t)P_t} - 1 \right] \times \frac{D_{t+1}}{P_{t+1}} \div \frac{D_{t+1}}{P_{t+1}} (1 + G_t) \]

\[ = \left[ \frac{P_{t+1}D_{t+1}}{(1 + G_t)P_tP_{t+1}} - \frac{D_{t+1}}{P_{t+1}} \right] \div \frac{D_{t+1}}{P_{t+1}} (1 + G_t) \]

\[ = \left[ \frac{D_t}{P_t} - \frac{D_{t+1}}{P_{t+1}} \right] \div \frac{D_{t+1}}{P_{t+1}} (1 + G_t) \]

\[ R \{ \Delta[(r - g)/(1 + g)] \}_t = \frac{\left[ \frac{D_t}{P_t} - \frac{D_{t+1}}{P_{t+1}} \right]}{D_{t+1}} (1 + G_t) \]  \hspace{1cm} (6)
Table 1
Data sources and calculation of returns

<table>
<thead>
<tr>
<th>Data item</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity yield and index value</td>
<td>1921-34: index constructed by Barclays Capital, in Equity-Gilt Study 2009 (EGS)</td>
</tr>
<tr>
<td></td>
<td>1935-62: FT 30 index, from EGS</td>
</tr>
<tr>
<td></td>
<td>1963-08: FT All-Share index, from Thomson Datastream</td>
</tr>
<tr>
<td>2.5% consols yield and price</td>
<td>1921-68: Capie &amp; Webber (2005)</td>
</tr>
<tr>
<td></td>
<td>1969-08: Thomson Datastream</td>
</tr>
<tr>
<td>Index-linked gilts</td>
<td>Thomson Datastream</td>
</tr>
<tr>
<td>Equity return</td>
<td>Inferred from change in yield(^1,2)</td>
</tr>
<tr>
<td>Return on 2.5% consols</td>
<td>Inferred from change in yield(^1)</td>
</tr>
<tr>
<td>Annual change in nominal dividend</td>
<td>Inferred from index value and yield(^3)</td>
</tr>
<tr>
<td>Annual inflation</td>
<td>From EGS</td>
</tr>
</tbody>
</table>

Notes

\(^1\) To ensure that our decompositions explain the relevant returns without error, we work with the returns on equity and bonds inferred from the annual changes in yield. These returns differ very slightly from the returns as recorded in the Equity-Gilt Study (Barclays Capital, 2009), presumably because of rounding in the yields. The formula for inferring a return \(R_t\) is \(R_t = (Y_t/Y_{t+1} - 1)100 + Y_t\), where the yield \(Y_t = D_t/P_t\).

\(^2\) There is a substantial difference between the return implied by the change in the yield and the return recorded in EGS for the years 1962 and 1997. These differences are caused by discontinuities in the yield figures, in 1962 because of the switch from the FT30 index to the FT All-Share index, and in 1997 because a change in the taxation of dividends. Adjustments to the yields are needed for these two years in order that the change in the yield produces the correct return on equity.

\(^3\) The formula is \(G_t = [(Y_{t+1} \times Index_{t+1})/(Y_t \times Index_t) - 1]100\).
Table 2
Results with inferred (variable) expected equity premium

The returns and premia are arithmetic means of annual returns and premia for the period 1921-2008. The formulae are explained in Section 2.2.

<table>
<thead>
<tr>
<th></th>
<th>Return or premium %</th>
<th>Standard deviation %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equity return</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual return, $R_t$</td>
<td>12.6</td>
<td>23.9</td>
</tr>
<tr>
<td>Of which, return due to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected return, $r_t$</td>
<td>9.7</td>
<td>3.0</td>
</tr>
<tr>
<td>Unexpected dividend growth, $R(\Delta \text{div})_t$</td>
<td>1.0</td>
<td>7.6</td>
</tr>
<tr>
<td>Change in $r_t - g_t$, $R{\Delta[(r - g)/(1 + g)]}_t$</td>
<td>1.9</td>
<td>22.9</td>
</tr>
<tr>
<td>Of which, return due to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in expected real rate of interest, $R(\Delta r_{Freal})_t$</td>
<td>0.7</td>
<td>7.6</td>
</tr>
<tr>
<td>Change in inflation risk premium, $R(\Delta irp)_t$</td>
<td>0.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Change in equity risk premium, $R(\Delta erp)_t$</td>
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<td>22.7</td>
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<tr>
<td>Change in expected real growth rate of dividends, $R(\Delta g_{real})_t$</td>
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<td>11.4</td>
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<tr>
<td><strong>Return on consols</strong></td>
<td></td>
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<tr>
<td>Actual return, $R_{F,t}$</td>
<td>7.6</td>
<td>14.2</td>
</tr>
<tr>
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<td></td>
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<tr>
<td>Yield on consols ( = expected return, $r_{F,t}$)</td>
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<td>3.4</td>
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<td>Change in $r_{F,t}$, $R_{F}(\Delta r_{F})_t$</td>
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<td>13.3</td>
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<td></td>
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<tr>
<td>Change in expected real rate of interest, $R_{F}(\Delta r_{Freal})_t$</td>
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<td>5.6</td>
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<tr>
<td>Change in expected rate of inflation, $R_{F}(\Delta i)_t$</td>
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<td>8.7</td>
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<tr>
<td>Change in inflation risk premium, $R_{F}(\Delta irp)_t$</td>
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<td>2.2</td>
</tr>
<tr>
<td><strong>Equity risk premium</strong></td>
<td></td>
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</tr>
<tr>
<td>Ex post premium, $R_t - R_{F,t}$</td>
<td>4.9</td>
<td>21.5</td>
</tr>
<tr>
<td>Of which</td>
<td></td>
<td></td>
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<tr>
<td>Expected premium, $r_t - R_{F,t}$</td>
<td>3.3</td>
<td>1.8</td>
</tr>
<tr>
<td>Unexpected premium, $R_t - R_{F,t} - (r_t - r_{F,t})$</td>
<td>1.7</td>
<td>21.0</td>
</tr>
<tr>
<td>Of which, premium due to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unexpected dividend growth, $R(\Delta \text{div})_t$</td>
<td>1.0</td>
<td>7.6</td>
</tr>
<tr>
<td>Change in expected real rate of interest, $R(\Delta r_{Freal})<em>t - R</em>{F}(\Delta r_{Freal})_t$</td>
<td>-0.1</td>
<td>3.3</td>
</tr>
<tr>
<td>Change in expected rate of inflation, $R(\Delta i)<em>t - R</em>{F}(\Delta i)_t$</td>
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<td>8.7</td>
</tr>
<tr>
<td>Change in inflation risk premium, $R(\Delta irp)<em>t - R</em>{F}(\Delta irp)_t$</td>
<td>0.0</td>
<td>1.8</td>
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<tr>
<td>Change in equity risk premium, $R(\Delta erp)_t$</td>
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<td>22.7</td>
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<tr>
<td>Change in expected real growth rate of dividends, $R(\Delta g_{real})_t$</td>
<td>-0.1</td>
<td>11.4</td>
</tr>
</tbody>
</table>
Table 3
Results with expected equity premium fixed at 3.3%

The returns are arithmetic means of annual returns for the period 1921-2008. The equity returns are simulated using a fixed expected premium of 3.3% and the returns implied by the actual values of the other variables that affect equity value. The balancing term arises because of the use of a fixed expected premium, and is defined in Section 4.2.

<table>
<thead>
<tr>
<th>Fixed-premium equity return</th>
<th>Return %</th>
<th>Standard deviation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated return, SimR&lt;sub&gt;t&lt;/sub&gt;</td>
<td>13.4</td>
<td>28.1</td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>Expected return, Simr&lt;sub&gt;t&lt;/sub&gt;</td>
<td>9.7</td>
<td>3.4</td>
</tr>
<tr>
<td>Unexpected dividend growth, R(SimΔdiv)&lt;sub&gt;t&lt;/sub&gt;</td>
<td>1.0</td>
<td>7.5</td>
</tr>
<tr>
<td>Change in Simr&lt;sub&gt;t&lt;/sub&gt; – g&lt;sub&gt;t&lt;/sub&gt;, R{Δ[Simr&lt;sub&gt;t&lt;/sub&gt; – g&lt;sub&gt;t&lt;/sub&gt;]/(1 + g&lt;sub&gt;t&lt;/sub&gt;)}&lt;sub&gt;t&lt;/sub&gt;</td>
<td>2.7</td>
<td>26.6</td>
</tr>
<tr>
<td>Of which, return due to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in expected real rate of interest, SimR(Δr&lt;sub&gt;real&lt;/sub&gt;)&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.7</td>
<td>7.1</td>
</tr>
<tr>
<td>Change in inflation risk premium, SimR(Δirp)&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.1</td>
<td>3.3</td>
</tr>
<tr>
<td>Change in expected real growth rate of dividends, SimR(Δg&lt;sub&gt;real&lt;/sub&gt;)&lt;sub&gt;t&lt;/sub&gt;</td>
<td>1.9</td>
<td>22.3</td>
</tr>
<tr>
<td>Balancing term</td>
<td>0.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Table 4
Volatilities of simulated fixed-premium equity returns under forecasts of differing variability

The table shows standard deviations of simulated fixed-premium equity returns, estimated under a range of assumptions about the volatilities of the expected real interest rate, $r_{Freal,t}$, and the expected real growth rate of dividends, $g_{real,t}$. The assumed changes in $r_{Freal,t}$, $\Delta(r_{Freal})^*$, are calculated from the formula $\Delta(r_{Freal})^* = \text{av}(r_{Freal,t}) + x[\Delta(r_{Freal,t}) - \text{av}(r_{Freal,t})]$, where $\text{av}(r_{Freal,t})$ is the arithmetic mean value of $r_{Freal,t}$ during the sample period, and $x$ is set at a value between 1 and 0 for each case. The same applies for $g_{real,t}$. The values of the two variables with $x = 1$ for each is the base case, reported in detail in Table 3. The values with $x = 0$ is the case with constant dividend yield.

<table>
<thead>
<tr>
<th>standard deviation of $g_{real,t}$ times</th>
<th>1.0</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>28.1</td>
<td>24.7</td>
<td>23.5</td>
<td>20.6</td>
<td>19.3</td>
<td>18.2</td>
</tr>
<tr>
<td>0.8</td>
<td>20.5</td>
<td>18.9</td>
<td>17.6</td>
<td>16.5</td>
<td>15.6</td>
<td>14.9</td>
</tr>
<tr>
<td>0.6</td>
<td>16.2</td>
<td>15.2</td>
<td>14.3</td>
<td>13.5</td>
<td>12.9</td>
<td>12.4</td>
</tr>
<tr>
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<td>13.5</td>
<td>12.7</td>
<td>12.1</td>
<td>11.5</td>
<td>11.0</td>
<td>10.6</td>
</tr>
<tr>
<td>0.2</td>
<td>11.9</td>
<td>11.3</td>
<td>10.7</td>
<td>10.2</td>
<td>9.8</td>
<td>9.4</td>
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<tr>
<td>0.0</td>
<td>11.3</td>
<td>10.7</td>
<td>10.2</td>
<td>9.7</td>
<td>9.3</td>
<td>8.9</td>
</tr>
</tbody>
</table>
Figure 1

Estimate of expected real dividend growth rate, 1921-2008 (% pa)

Figure 2

Estimate of expected real interest rate, 1921-2008 (% pa)
Figure 3

Actual equity return less return due to change in expected premium (first bar), and return due to change in expected premium (second bar) (%), 1960-2008

Figure 4

Actual equity return (first bar) and simulated return with expected premium fixed at 3.3% (second bar) (%), 1960-2008
References


Barclays Capital (2009), Equity-Gilt Study.


